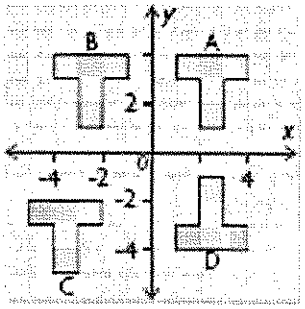
Chapter 5: Applying Quadratic Models

Getting Started, page 246

1. a) While iii) and v) are possible, because sliding each point on a shape and reflecting a shape are transformations, the best choice is ii): the result of moving or changing the size of a shape according to a rule.
b) A translation is the result of sliding each point on a shape the same distance. The correct choice is iii).
c) A reflection is the result of reflecting a shape to produce a mirror image of the shape; v) is correct.
d) A parabola is the graph of a quadratic relation. The correct choice is vi).
e) A vertex is a point that relates to the maximum or minimum value of a quadratic relation; i) is correct.
f) When a quadratic relation is in factored form, it is in the form $y = a(x - r)(x - s)$; iv) is correct.
2. a) Figure C is the only figure that is a translation of figure A. Each square of figure A is translated 6 units left and 6 units down.
b) Figure D is the only figure that is a reflection of figure A in the x-axis. Each square of Figure A is reflected in x-axis.
c) Figure B is the only figure that is a reflection of figure A in the y-axis. Each square of Figure A is reflected in x-axis.



3. a) for $x = -4$,

$$y = x^2 + 2x + 5$$
$$y = (-4)^2 + 2(-4) + 5$$
$$y = 16 - 8 + 5$$
$$y = 13$$

b) for $x = 7$,

$$y = x^2 - 3x - 28$$
$$y = 7^2 - 3(7) - 28$$
$$y = 49 - 21 - 28$$
$$y = 0$$

4. The zeros of a relation occur when $y = 0$.
a) For $y = (x + 5)(x - 3)$, $y = 0$ when $x = -5$ and $x = 3$, so the zeros are -5 and 3.
The equation of the axis of symmetry is

$$x = \frac{(-5 + 3)}{2}$$
$$x = -1$$

To determine the vertex, substitute -1 into the equation:

$$y = (x + 5)(x - 3)$$
$$y = (-1 + 5)(-1 - 3)$$
$$y = (4)(-4)$$
$$y = -16$$

The vertex is $(-1, -16)$.

b) For $y = 2(x - 4)(x + 1)$, $y = 0$ when $x = 4$ and $x = -1$, so the zeros are -1 and 4.
The equation of the axis of symmetry is

$$x = \frac{-1 + 4}{2}$$
$$x = 1.5$$

To determine the vertex, substitute 1.5 into the equation:

$$y = 2(x - 4)(x + 1)$$
$$y = 2(1.5 - 4)(1.5 + 1)$$
$$y = 2(-2.5)(2.5)$$
$$y = -12.5$$

The vertex is $(1.5, -12.5)$.
c) For $y = -4x(x + 3)$, $y = 0$ when $x = 0$ and $x = -3$, so the zeros are 0 and -3.
The equation of the axis of symmetry is

$$x = \frac{-3 + 0}{2}$$
$$x = -1.5$$

To determine the vertex, substitute -1.5 into the equation:

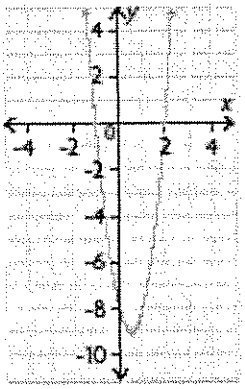
$$y = -4x(x + 3)$$
$$y = -4(-1.5)(-1.5 + 3)$$
$$y = -4(-1.5)(1.5)$$
$$y = 9$$

The vertex is $(-1.5, 9)$.
5. a) A translation of 2 units left decreases the x-coordinate by 2 and a translation of 5 units up increases the y-coordinate by 5, so the coordinates of A' are $(3 - 2, 4 + 5) = (1, 9)$.
b) A translation of 4 units right increases the x-coordinate by 4 and a translation of 3 units down decreases the y-coordinate by 3, so the coordinates of B' are $(-1 + 4, -5 - 3) = (3, -8)$.
c) A translation of 2 units left decreases the x-coordinate by 2 and a translation of 7 units up increases the y-coordinate by 7, so the coordinates of C' are $(2 - 2, -7 + 7) = (0, 0)$.
d) A reflection in the x-axis means the x-coordinate stays the same and the y-coordinate becomes its opposite. The coordinates of D are $(3, -5)$, so the coordinates of D' are $(3, 5)$.
6. a) The coordinates of $\triangle PQR$ are $P(-1, 3)$, $Q(0, 6)$, $R(3, 4)$. The coordinates of $\triangle P'Q'R'$ are $P'(4, 0)$, $Q'(5, 3)$, $R'(8, 1)$. Each image x-coordinate is 5 greater than the original and each image y-coordinate is 3 less than the original, so $\triangle PQR$ was translated 5 units right and 3 units down to get $\triangle P'Q'R'$.
b) The coordinates of $\triangle PQR$ are $P(-1, 3)$, $Q(0, 6)$, $R(3, 4)$. The coordinates of $\triangle P''Q''R''$ are $P''(2, -3)$, $Q''(3, 0)$, $R''(6, -2)$. Each image x-coordinate is 3 greater than the original and each image y-coordinate is 6 less than the original, so $\triangle PQR$ was translated 3 units right and 6 units down to get $\triangle P''Q''R''$.
c) The coordinates of $\triangle P'Q'R'$ are $P'(4, 0)$, $Q'(5, 3)$, $R'(8, 1)$. The coordinates of $\triangle P''Q''R''$ are $P''(2, -3)$, $Q''(3, 0)$, $R''(6, -2)$. Each image x-coordinate is 2 less than the original and each image y-coordinate is 3 less than the original, so $\triangle P'Q'R'$ was translated 2 units left and 3 units down to get $\triangle P''Q''R''$.

7. a) The graph crosses the x-axis at $x = -1$ and $x = 2$, so the zeros are -1 and 2. The equation of the axis of symmetry is

$$x = \frac{-1+2}{2}$$
$$x = 0.5$$

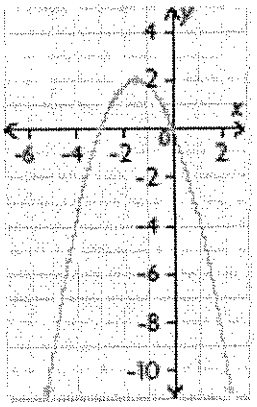
By examining the graph for the lowest point, the vertex is $(0.5, -9)$.



b) The graph crosses the x-axis at $x = -3$ and $x = 0$, so the zeros are -3 and 0. The axis of symmetry is:

$$x = \frac{-3+0}{2}$$
$$x = -1.5$$

By examining the graph for the highest point, the vertex is $(-1.5, 2)$.



8. a) $y = (x + 4)(x + 5)$
$y = x^2 + 5x + 4x + 20$
$y = x^2 + 9x + 20$

b) $y = (2x - 3)(x + 2)$
$y = 2x^2 + 4x - 3x - 6$
$y = 2x^2 + x - 6$

c) $y = -3(x - 4)(x + 7)$
$y = -3(x^2 + 7x - 4x - 28)$
$y = -3(x^2 + 3x - 28)$
$y = -3x^2 - 9x + 84$

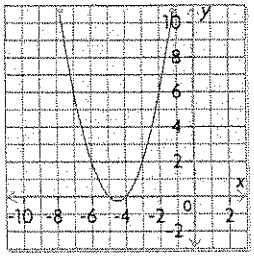
d) $y = (x + 5)^2$
$y = (x + 5)(x + 5)$
$y = x^2 + 5x + 5x + 25$
$y = x^2 + 10x + 25$

9. a) The relation is quadratic and a is positive, so this is a parabola that opens upward. The zeros are $x = -4$ and $x = -5$, so the graph goes through those points on the x-axis. The equation of the axis of symmetry is:

$$x = \frac{-4-5}{2}$$
$$x = -4.5$$

Substitute $x = -4.5$ into the equation:
$y = (-4.5 + 4)(-4.5 + 5)$
$y = (-0.5)(0.5)$
$y = -0.25$

The vertex is $(-4.5, -0.25)$.

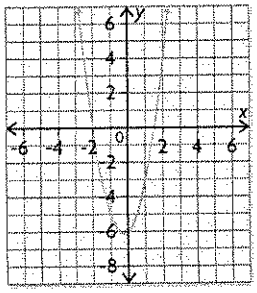


b) The relation is quadratic and a is positive, so this is a parabola that opens upward. The zeros are $x = 1.5$ and $x = -2$, so the graph goes through those points on the x-axis. The equation of the axis of symmetry is:

$$x = \frac{-2+1.5}{2}$$
$$x = -0.25$$

Substitute $x = -0.25$ into the equation:
$y = 2x^2 + x - 6$
$y = 2(-0.25)^2 + (-0.25) - 6$
$y = -6.125$

The vertex is $(-0.25, -6.125)$.

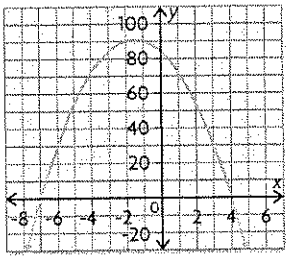


c) The relation is quadratic and a is negative, so this is a parabola that opens downward. The zeros are $x = 4$ and $x = -7$, so the graph goes through those points on the x-axis. The equation of the axis of symmetry is:

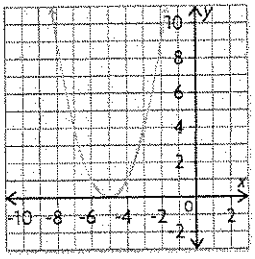
$$x = \frac{-7+4}{2}$$
$$x = -1.5$$

Substitute $x = -1.5$ into the equation:
$y = -3x^2 - 9x + 84$
$y = -3(-1.5)^2 - 9(-1.5) + 84$
$y = 90.75$

The vertex is $(-1.5, 90.75)$.



d) The relation is quadratic and a is positive, so this is a parabola that opens downward. There is just one zero, $x = -5$, so the graph touches this point on the x-axis. Since there is just one zero, the vertex must be $(-5, 0)$.



10. $y = 2x^2 - 4x - 48$
$y = 2(x^2 - 2x - 24)$
$y = 2(x + 4)(x - 6)$
This is a parabola that opens upward, so the minimum value occurs at the vertex. The equation of the axis of symmetry is:

$$x = \frac{-4 + 6}{2} = 1$$

So, the minimum value occurs when $x = 1$:
$y = 2(1)^2 - 4(1) - 48$
$y = 2 - 4 - 48$
$y = -50$

11.

Definition: A relation that can be described by an equation with a polynomial whose highest degree is 2	Special Properties: The graph has a vertical line of symmetry. The graph also has a single minimum or maximum value.
Examples: $y = x^2 + 9x + 2$ $y = 2(x + 4)(x - 6)$ $y = 4(x + 2)^2 - 3$	Non-examples: $y = x + 9$ $y = x^3 + 9x + 3$ $y = \sqrt{x}$

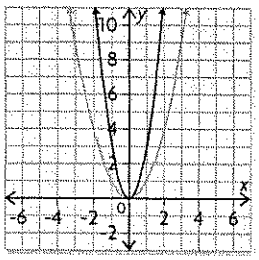
Quadratic Relation

Lesson 5.1: Stretching/Reflecting Quadratic Relations, page 256

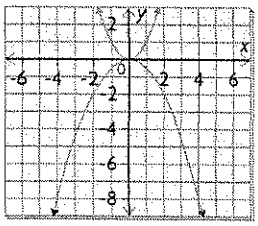
1. a) For $y = 4x^2$, $a = 4$, which is positive and greater than 1, so the graph is a parabola that opens upward and is stretched vertically. Graphs ii) and iii) are eliminated, because they open downward. Graph iv) is the correct answer because it is the only graph that opens upward and is narrower than $y = x^2$.
b) For $y = -3x^2$, $a = -3$, which is negative and less than -1, so the graph is a parabola that is stretched vertically and reflected across the x-axis (opens downward). Graphs i) and iv) are eliminated, because they open upward. Graph iii) is the correct answer because only this graph opens downward and is narrower than $y = x^2$.
c) For $y = \frac{2}{3}x^2$, $a = \frac{2}{3}$, which is positive and less than 1, so the graph is a parabola that opens upward and is compressed vertically. Graph i) is the correct answer because only this graph opens upward and is wider than $y = x^2$.
d) For $y = -0.4x^2$, $a = -0.4$, which is negative and greater than -1, so the graph is a parabola that is reflected across the x-axis and is compressed vertically. Graph ii) is the correct answer because only this graph opens downward and is wider than $y = x^2$.
2. a) For $a = 5$ and $x = 1$,
$y = ax^2$
$y = 5(1)^2$
$y = 5$
The coordinates of the transformed point are (1, 5).
b) For $a = -3$ and $x = -2$,
$y = ax^2$
$y = -3(-2)^2$
$y = -3(4)$
$y = -12$
The coordinates of the transformed point are (–2, –12).
c) For $a = -0.6$ and $x = 5$,
$y = ax^2$
$y = -0.6(5)^2$
$y = -0.6(25)$
$y = -15$
The coordinates of the transformed point are (5, –15).
d) For $a = \frac{1}{2}$ and $x = -4$,

$$y = ax^2$$
$$= \frac{1}{2}(-4)^2$$
$$= \frac{1}{2}(16)$$
$$= 8$$

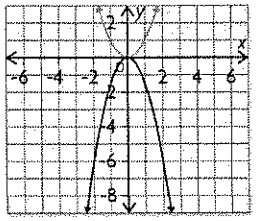
The coordinates of the transformed point are (–4, 8).
3. a) Since the graphs of these equations are narrower than the graph of $y = x^2$, then the coefficients of x^2 must be greater than 1. For example, $y = 4x^2$ and $y = 1.01x^2$.
b) Since the graphs of these equations are compressed vertically from the graph of $y = -x^2$, then it must be that $a < 1$. For example, $y = 0.5x^2$, $y = -0.1x^2$.
c) Since the graphs of these equations are narrower than the graph of $y = 3x^2$, then it must be that $a < -3$. For example, $y = -3.1x^2$, $y = -6x^2$.
4. a) For $y = 3x^2$, $a = 3$, which is positive and greater than 1, so the graph is a parabola that opens upward and is narrower than $y = x^2$. Point (1, 1) is transformed to (1, 3) and point (–1, 1) is transformed to (–1, 3).



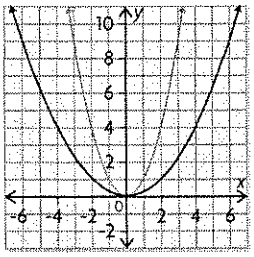
b) For $y = -0.5x^2$, $a = -0.5$, which is negative and greater than -1, so the graph is a parabola that opens downward and is wider than $y = x^2$. Point (1, 1) is transformed to (1, –0.5) and point (–1, 1) is transformed to (–1, 0.5).



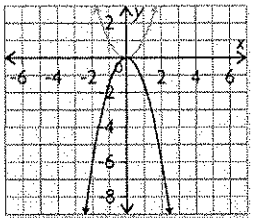
c) For $y = -2x^2$, $a = -2$, which is negative and less than -1, so the graph is a parabola that opens downward and is narrower than $y = x^2$. Point (1, 1) is transformed to (1, –2) and point (–1, 1) is transformed to (–1, –2).



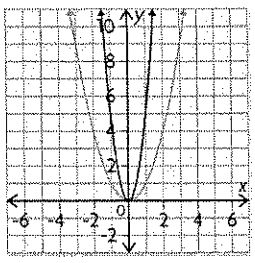
d) For $y = \frac{1}{4}x^2$, $a = \frac{1}{4}$, which is positive and less than 1, so the graph is a parabola that opens upward and is wider than $y = x^2$. Point (1, 1) is transformed to (1, 0.25) and point (–1, 1) is transformed to (–1, 0.25).



e) For $y = -\frac{3}{2}x^2$, $a = -\frac{3}{2}$, which is negative and is less than -1, so the graph is a parabola that opens downward and is narrower than $y = x^2$. Point (1, 1) is transformed to (1, –1.5) and point (–1, 1) is transformed to (–1, –1.5).



f) For $y = 5x^2$, $a = 5$, which is positive and is greater than 1, so the graph is a parabola that opens upward and is narrower than $y = x^2$. Point (1, 1) is transformed to (1, 5) and point (–1, 1) is transformed to (–1, 5).



5. a) The new graph opens upward, with a vertex of (0, 0). The point (1, 1) was transformed to (1, 4), so the graph $y = x^2$ was vertically stretched by a factor of 4. The equation of the new graph is $y = 4x^2$.
b) The new graph opens downward, with a vertex of (0, 0). The point (1, 1) was transformed to (1, –0.5), so the graph $y = x^2$ was vertically compressed by a factor of $\frac{1}{2}$ and reflected in the x-axis. The equation of the new graph is $y = -0.5x^2$.
c) The new graph has the same vertex, (0, 0). The point (1, 1) was transformed into (1, –2.5). So, the graph $y = x^2$ was vertically stretched by a factor of 2.5 and reflected in the x-axis. The equation of the new graph is $y = -2.5x^2$.
d) The new graph has the same vertex, (0, 0). The point (1, 1) was transformed into (1, 0.25), so the graph $y = x^2$ was vertically compressed by a factor of $\frac{1}{4}$. The equation of the new graph is $y = \frac{1}{4}x^2$.

6. For example, (2, –0.5) lies on the arch, so choose this point and substitute it into $y = ax^2$, and solve for a.

$$y = ax^2$$
$$-0.5 = a(2)^2$$
$$-0.5 = 4a$$
$$a = \frac{-0.5}{4}$$
$$a = -0.125$$

A quadratic model for the arch is $y = -0.125x^2$

7. a) For example, point (–3, –5) lies close to the parabola so choose this point and substitute it into $y = ax^2$, and solve for a.

$$y = ax^2$$
$$-5 = a(-3)^2$$
$$-5 = 9a$$
$$a = -\frac{5}{9}$$

A quadratic model for the arch is

$$y = -\frac{5}{9}x^2$$

b) For example, point (–4, –3) lies close to the arch, so choose this point and substitute it into $y = ax^2$, and solve for a.

$$y = ax^2$$
$$-3 = a(-4)^2$$
$$-3 = 16a$$
$$a = \frac{-3}{16}$$

A quadratic model for the arch is

$$y = -\frac{3}{16}x^2$$

8. a) The graph of $y = x^2$ must be stretched vertically by a factor of 4.
For $a = 4$ and $x = 2$,

$$y = ax^2$$
$$y = 4(2)^2$$
$$y = 16$$

The coordinates of the transformed point are (2, 16).

b) The graph of $y = x^2$ is reflected in the x-axis and compressed vertically by a factor of $\frac{2}{3}$. For $a = -\frac{2}{3}$ and $x = 2$,

$y = ax^2$

$y = -\frac{2}{3}(2)^2$

$y = -\frac{2}{3}(4)$

$y = -\frac{8}{3}$

The coordinates of the transformed point are $\left(2, -\frac{8}{3}\right)$.

c) The graph of $y = x^2$ is compressed vertically by a factor of 0.25.
For $a = 0.25$ and $x = 2$,

$y = ax^2$

$y = 0.25(2)^2$

$y = 1$

The coordinates of the transformed point are (2, 1).
d) The graph of $y = x^2$ is reflected in the x-axis and stretched vertically by a factor of 5. For $a = -5$ and $x = 2$,

$y = ax^2$

$y = -5(2)^2$

$y = -20$

The coordinates of the transformed point are (2, –20).
e) Reflect the graph of $y = x^2$ in the x-axis.
For $a = -1$ and $x = 2$,

$y = ax^2$

$y = -1(2)^2$

$y = -4$

The coordinates of the transformed point are (2, –4).
f) Compress the graph of $y = x^2$ vertically by a factor of $\frac{1}{5}$. For $a = \frac{1}{5}$ and $x = 2$,

$y = ax^2$

$= \frac{1}{5}(2)^2$

$= \frac{1}{5}(4)$

$= \frac{4}{5}$

The coordinates of the transformed point are $\left(2, \frac{4}{5}\right)$.

9. For example, on my grid, the lower outline of the bridge went through point (3, –1). So,

$y = ax^2$

$-1 = a(3)^2$

$-1 = 9a$

$a = -\frac{1}{9}$

$y = -\frac{1}{9}x^2$

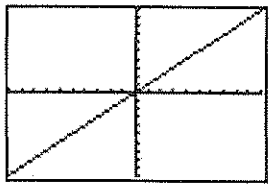
10. Disagree. The graph of $y = ax^2$ is congruent to the graph of $y = x^2$ when $a = -1$ and $a = 1$.

11.

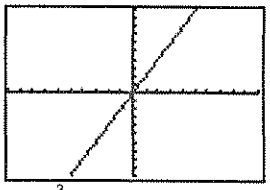
Equation	Direction of Opening (upward/ downward)	Description of Transformation (stretch/ compress)	Shape of Graph Compared with Graph of $y = x^2$ (wider/narrower)
$y = 5x^2$	upward	stretch	narrower
$y = 0.25x^2$	upward	compress	wider
$y = -\frac{1}{3}x^2$	downward	compress	wider
$y = -8x^2$	downward	stretch	narrower

12. a) All the y-coordinates are multiplied by a negative number. So, all the points on the graph $y = ax^2$ are reflected in the x-axis, causing it to open downward.
b) The y-coordinates of the points on the graph are multiplied by a fraction whose magnitude is less than 1, so the points are moved toward the x-axis, making the parabola wider.
c) Since the y-coordinate of the vertex is 0, and multiplying 0 by any number results in a value of 0, the vertex is not affected.
13. For example, I used my graphing calculator to graph each type of equation for $a = 1$, 2, and 0.5.

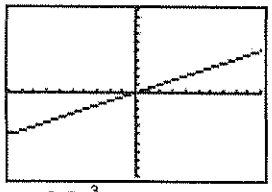
$y = x$



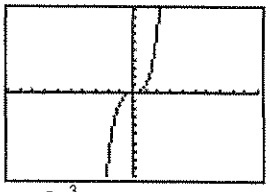
$y = 2x$



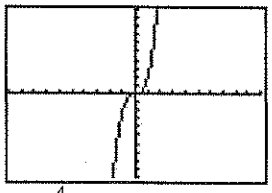
$y = 0.5x$



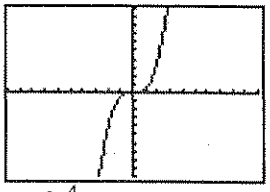
$y = x^3$



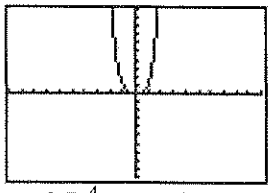
$y = 0.5x^3$



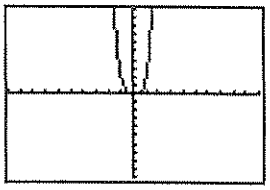
$y = 2x^3$



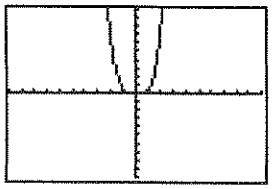
$y = x^4$



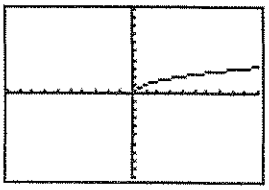
$y = 2x^4$



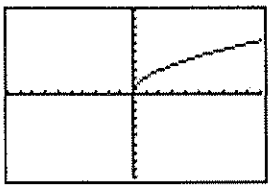
$y = 0.5x^4$



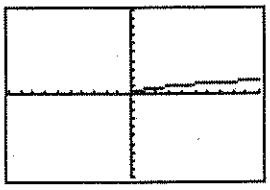
$y = x^{\frac{1}{2}}$



$y = 2x^{\frac{1}{2}}$



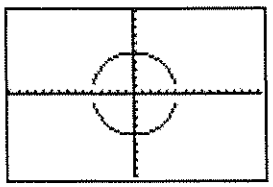
$y = 0.5x^{\frac{1}{2}}$



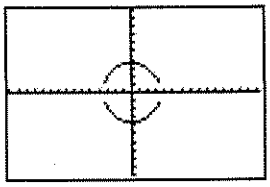
In each case, the effect is the same on all graphs.

14. a) I rewrote the equation $ax^2 + ay^2 = r^2$ in the form $y = \sqrt{\frac{r^2 - ax^2}{a}}$ and $y = -\sqrt{\frac{r^2 - ax^2}{a}}$. Then I chose 5 as the radius of the circle to get $y = \sqrt{\frac{5^2 - ax^2}{a}}$ and $y = -\sqrt{\frac{5^2 - ax^2}{a}}$. I graphed these equations on my calculator for $a = 1$, 2, and 0.5.

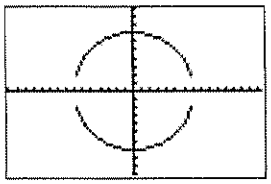
For $a = 1$:



For $a = 2$:



For $a = 0.5$:

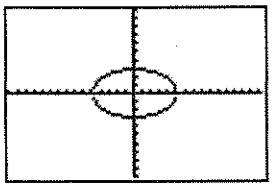


As a increases, the radius of the circle decreases, and as a decreases, the radius increases.

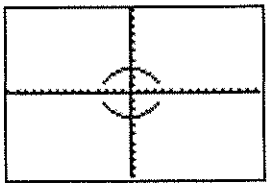
b) The graph of $ax^2 + by^2 = r^2$ is an ellipse. I rewrote the equation $ax^2 + by^2 = r^2$ in the form $y = \sqrt{\frac{r^2 - ax^2}{b}}$ and $y = -\sqrt{\frac{r^2 - ax^2}{b}}$. Then I chose $r = 5$ as the radius of the ellipse to get $y = \sqrt{\frac{5^2 - ax^2}{b}}$ and $y = -\sqrt{\frac{5^2 - ax^2}{b}}$.

I graphed these equations for $a = 1$, 2, and 0.5 and $b = 3$.

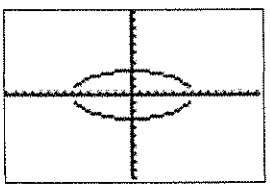
For $a = 1$ and $b = 3$:



For $a = 2$ and $b = 3$:



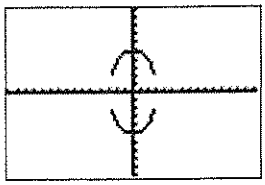
For $a = 0.5$ and $b = 3$:



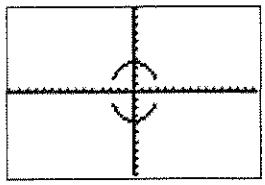
As the value of a increases, the width of the ellipse along the x-axis decreases. As the value of a decreases, the width of the ellipse along the x-axis increases.

I graphed these equations for $a = 3$ and $b = 1$, 2, and 0.5.

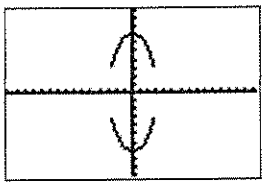
For $a = 3$ and $b = 1$:



For $a = 3$ and $b = 2$:



For $a = 3$ and $b = 0.5$:



As the value of b increases, the width of the ellipse along the y-axis decreases. As the value of b decreases, the width of the ellipse along the y-axis increases. The greater the difference between a and b, the narrower the ellipse.

If $b > a$, then the circle becomes an ellipse that is wider along the x-axis. If $a > b$, then the circle becomes an ellipse that is wider along the y-axis.

Lesson 5.2: Exploring Translations of Quadratic Relations, page 259

1. a) The parabola moves 3 units right and 0 units up or down, so $h = 3$ and $k = 0$. The equation is:

$y = (x - h)^2 + k$
$y = (x - 3)^2 + 0$
$y = (x - 3)^2$

b) The parabola moves 0 units left or right and 4 units down, so $h = 0$ and $k = -4$. The equation is:

$y = (x - h)^2 + k$
$y = (x - 0)^2 + (-4)$
$y = x^2 - 4$

c) The parabola moves 2 units left and 0 units up or down, so $h = -2$ and $k = 0$. The equation is:

$y = (x - h)^2 + k$
$y = [x - (-2)]^2 + 0$
$y = (x + 2)^2$

d) The parabola moves 0 units left or right and 5 units up, so $h = 0$ and $k = 5$. The equation is:

$y = (x - h)^2 + k$
$y = (x - 0)^2 + 5$
$y = x^2 + 5$

e) The parabola moves 6 units left and 7 units down, so $h = -6$ and $k = -7$. The equation is:

$y = (x - h)^2 + k$
$y = [x - (-6)]^2 + (-7)$
$y = (x + 6)^2 - 7$

f) The parabola moves 2 units right and 5 units up, so $h = 2$ and $k = 5$. The equation is:

$y = (x - h)^2 + k$
$y = (x - 2)^2 + 5$

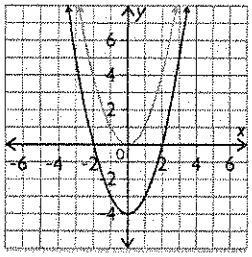
2. a) The graph of $y = (x - 2)^2 + 3$ is the graph of $y = x^2$ moved 2 units right and 3 units up. This is graph iii).

b) The graph of $y = (x + 2)^2 - 3$ is the graph of $y = x^2$ moved 2 units left and 3 units down. This is graph v).

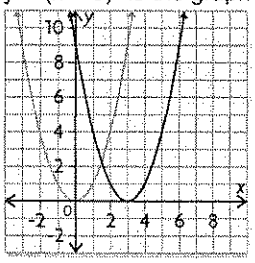
c) The graph of $y = (x + 3)^2 - 2$ is the graph of $y = x^2$ moved 3 units left and 2 units down. This is graph ii).

d) The graph of $y = (x - 3)^2 + 2$ is the graph of $y = x^2$ moved 3 units right and 2 units up. This is graph iv).

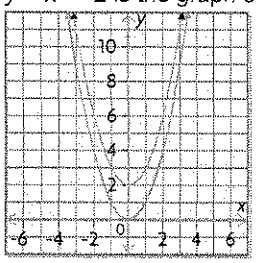
3. a) For $y = x^2 - 4$, $h = 0$ and $k = -4$. So, the graph of $y = x^2 - 4$ is the graph of $y = x^2$ moved 4 units down.



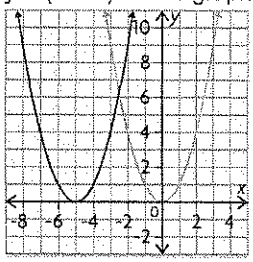
b) For $y = (x - 3)^2$, $h = 3$ and $k = 0$. So, the graph of $y = (x - 3)^2$ is the graph of $y = x^2$ moved 3 units right.



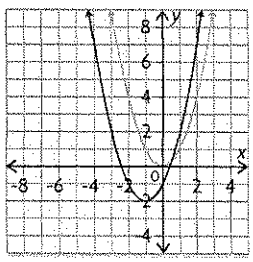
c) For $y = x^2 + 2$, $h = 0$ and $k = 2$. So, the graph of $y = x^2 + 2$ is the graph of $y = x^2$ moved 2 units up.



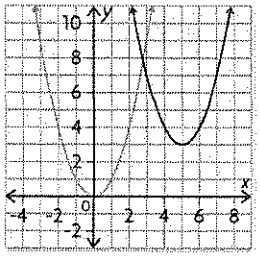
d) For $y = (x + 5)^2$, $h = -5$ and $k = 0$. So, the graph of $y = (x + 5)^2$ is the graph of $y = x^2$ moved 5 units left.



e) For $y = (x + 1)^2 - 2$, $h = -1$ and $k = -2$. So, the graph of $y = (x + 1)^2 - 2$ is the graph of $y = x^2$ moved 1 unit left and 2 units down.



f) For $y = (x - 5)^2 + 3$, $h = 5$ and $k = 3$. So, the graph of $y = (x - 5)^2 + 3$ is the graph of $y = x^2$ moved 5 units right and 3 units up, as shown.



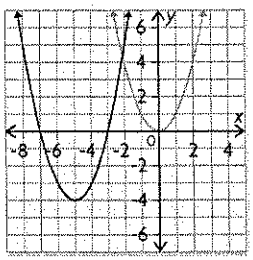
4. a) For $y = x^2 + 5$, $h = 0$, $k = 5$, and $a = 1$, so the graph of $y = x^2$ moves 5 units up.
b) For $y = (x - 3)^2$, $h = 3$, $k = 0$, and $a = 1$, so the graph of $y = x^2$ moves 3 units left.
c) For $y = -3x^2$, $h = 0$, $k = 0$, and $a = -3$, so the graph of $y = x^2$ is reflected in the axis and stretched vertically by a factor of 3.
d) For $y = (x + 7)^2$, $h = -7$, $k = 0$, and $a = 1$, so the graph of $y = x^2$ moves left 7 units.
e) For $y = \frac{1}{2}x^2$, $h = 0$, $k = 0$, and $a = \frac{1}{2}$, so the graph of $y = x^2$ is compressed vertically by a factor of $\frac{1}{2}$.
f) For $y = (x + 6)^2 + 12$, $h = -6$, $k = 12$, and $a = 1$, so the graph of $y = x^2$ moves left 6 units and up 12 units.
5. a) For $y = x^2 + 5$, $h = 0$ and $k = 5$, so the vertex is $(0, 5)$ and the equation of the axis of symmetry is $x = 0$.
b) For $y = (x - 3)^2$, $h = 3$ and $k = 0$, so the vertex is $(3, 0)$ and the equation of the axis of symmetry is $x = 3$.
c) For $y = -3x^2$, $h = 0$, $k = 0$, so the vertex is $(0, 0)$ and the equation of the axis of symmetry is $x = 0$.
d) For $y = (x + 7)^2$, $h = -7$ and $k = 0$, so the vertex is $(-7, 0)$ and the axis of symmetry is $x = -7$.
e) For $y = \frac{1}{2}x^2$, $h = 0$ and $k = 0$, so the vertex is $(0, 0)$ and the equation of the axis of symmetry is $x = 0$.
f) For $y = (x + 6)^2 + 12$, $h = -6$ and $k = 12$, so the vertex is $(-6, 12)$ and the axis of symmetry is $x = -6$.

Lesson 5.3: Graphing Quadratics in Vertex Form, page 263

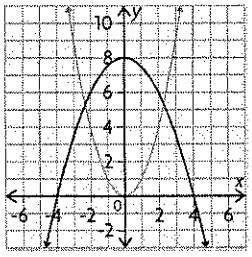
1. a) Shift the graph down 3 units, since $k = -3$.
b) Shift the graph left 5 units, since $h = -5$.
c) Reflect the graph in the x-axis, then compress it by a factor of $\frac{1}{2}$, since $a = -\frac{1}{2}$ and so $-1 < a < 0$.
d) First, stretch the result by a factor of 4, since $a = 4$ and so $a > 1$. Then, shift the graph left 2, since this relation has $h = -2$. Last, shift the result down 16, since $k = -16$.
2. a) i) The parabola opens up since $a > 0$.
ii) The vertex is $(0, -3)$ since $h = 0$ and $k = -3$.
iii) The axis of symmetry is $x = 0$ since $h = 0$.
b) i) The parabola opens up since $a > 0$.
ii) The vertex is $(-5, 0)$ since $h = -5$ and $k = 0$.
iii) The axis of symmetry is $x = -5$ since $h = -5$.
c) i) The parabola opens down since $a < 0$.
ii) The vertex is $(0, 0)$ since $h = 0$ and $k = 0$.
iii) The axis of symmetry is $x = 0$ since $h = 0$.

d) i) The parabola opens up since $a > 0$.
ii) The vertex is $(-2, -16)$ since $h = -2$ and $k = -16$.
iii) The axis of symmetry is $x = -2$ since $h = -2$.

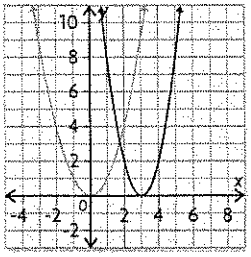
3. a) First, sketch $y = x^2$. Shift the graph left 5 units since $h = -5$ and down 4 units since $k = -4$.



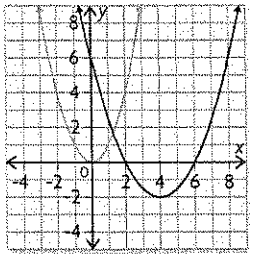
b) First, sketch $y = x^2$. Reflect the graph in the x-axis and compress the result by a factor of 0.5, since $a = -0.5$ and so $-1 < a < 0$. Shift the graph up 8 units, since $k = 8$.



c) First, sketch $y = x^2$. Stretch the graph by a factor of 2, since $a = 2$ and so $a > 1$. Shift the resulting graph right 3 units, since $h = 3$.



d) First, sketch $y = x^2$. Compress the graph by a factor of $\frac{1}{2}$, since $a = \frac{1}{2}$ and so $0 < a < 1$. Shift the graph right 4 units and down 2 units, since $h = 4$ and $k = -2$.

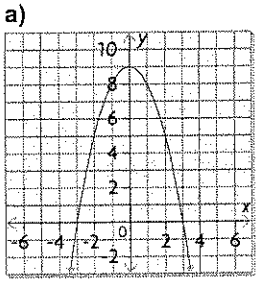


4. a) First, reflect the graph over the x-axis, since $a = -1$. Then, shift the graph up 9 units, since $k = 9$.
b) Shift the graph right 3 units, since $h = 3$.
c) Shift left 2 units, down 1 unit, since $h = -2$ and $k = -1$.
d) First, reflect the graph over the x-axis, since $a < 0$. Then, shift the graph down 6 units, since $k = -6$.
e) Reflect the graph in the x-axis and stretch it vertically by a factor of 2, since $a = -2$. Next, shift the graph right 4 units and up 16 units, since $h = 4$ and $k = 16$.

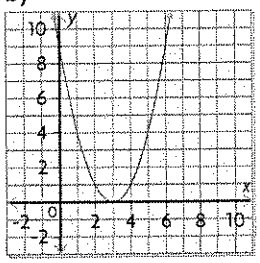
f) First, vertically compress the graph by a factor of $\frac{1}{2}$, since $a = \frac{1}{2}$. Then, shift the graph left 6 units and up 12 units, since $h = -6$ and $k = 12$.
g) First, reflect the graph over the x-axis and compress the graph vertically by a factor of $\frac{1}{2}$, since $a = -\frac{1}{2}$. Then, shift the graph left 4 units and down 7 units, since $h = -4$ and $k = -7$.
h) First, stretch the graph by a factor of 5, since $a = 5$. Then, shift the graph right 4 units and down 12 units, since $h = 4$ and $k = -12$.

5. The following graphs have been constructed according to the transformations listed in question 4. Determine any extra points required by producing a table of values.

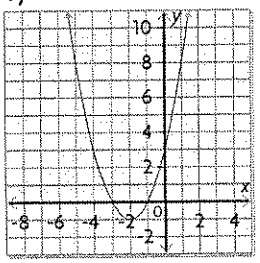
a)



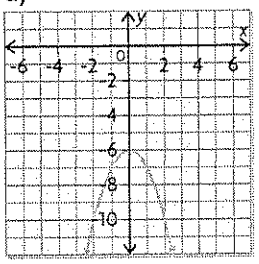
b)



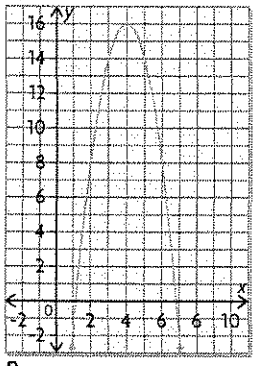
c)



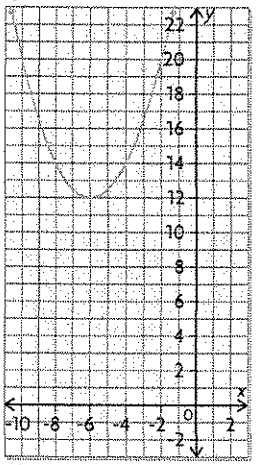
d)



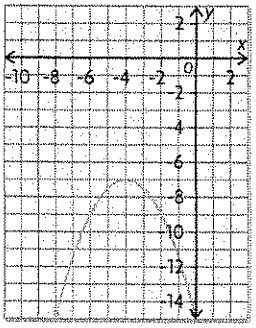
e)



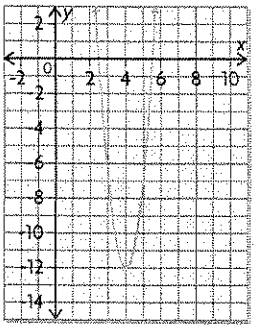
f)



g)



h)



6. a) Graph ii). For equation a), $k = -5$, so look for a parabola that has a vertex with a y-coordinate of -5. The parabolas in graphs ii) and vi) match this description. Equation a) also has $a > 0$; so, the parabola must open up. This eliminates graph vi). The correct answer is graph ii).

b) Graph v). For equation b), $h = 4$, so look for a parabola that has a vertex with an x-coordinate of 4. The only parabola that matches this description is graph v). The correct answer is graph v).

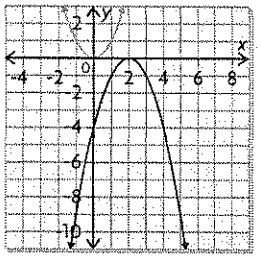
c) Graph iv). For equation c), $k = 5$, so look for a parabola that has a vertex with a y-coordinate of 5. Graph vi) is the only graph that matches this description. The correct answer is graph vi).

d) Graph vi). For equation d), $k = -5$, so look for a parabola that has a vertex with a y-coordinate of -5. Graphs ii) and vi) both match this description. Equation d) also has $a < 0$; so, the parabola must open down. This eliminates graph ii). The correct answer is graph vi).

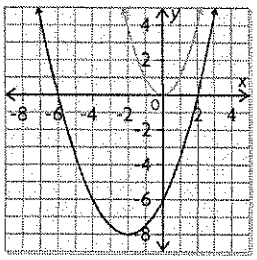
e) Graph i). For equation e), $h = 5$, so look for a parabola that has a vertex with an x-coordinate of 5. Graph i) is the only graph matches this description. The correct answer is graph i).

f) Graph iii). For equation f), $h = 0$, so look for a parabola that has a vertex with an x-coordinate of 0. Graph iii) is the only graph that matches this description. The correct answer is graph iii).

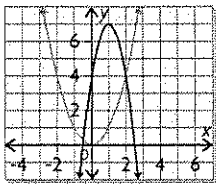
7. a) First, reflect the sketch of $y = x^2$ over the x-axis, since $a = -1$. Shift the resulting graph right 2 units, since $h = 2$.



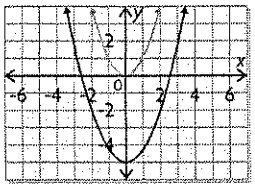
b) First, compress the sketch of $y = x^2$ by a factor of $\frac{1}{2}$, since $a = \frac{1}{2}$. Shift the resulting graph left 2 units and down 8 units, since $h = -2$ and $k = -8$.



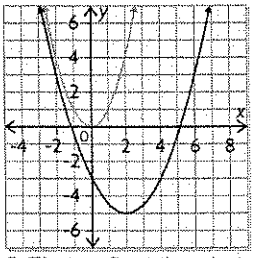
c) First, reflect the sketch of $y = x^2$ over the x-axis and stretch the result by a factor of 3, since $a = -3$. Shift the resulting graph right 1 unit and up 7 units, since $h = 1$ and $k = 7$.



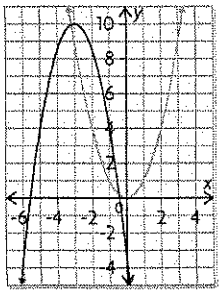
d) First, compress the sketch of $y = x^2$ by a factor of $\frac{3}{4}$, since $a = \frac{3}{4}$. Shift the resulting graph down 5 units, since $k = -5$.



e) First, compress the sketch of $y = x^2$ by a factor of $\frac{1}{2}$, since $a = \frac{1}{2}$. Shift the resulting graph right 2 units and down 5 units, since $h = 2$ and $k = -5$.



f) First, reflect the sketch of $y = x^2$ over the x-axis and stretch the result by a factor of 1.5, since $a = -1.5$. Shift the resulting graph left 3 units and up 10 units, since $h = -3$ and $k = 10$.



8. The table follows these explanations.

Row 1 has a stretch factor of 3 and no reflection in the x-axis, so $a = 3$. It has a vertex at $(2, -5)$, so $h = 2$ and $k = -5$. Its quadratic relation is $y = 3(x - 2)^2 - 5$.

Row 2 has $a = 4$; so, its stretch factor is 4, and there is no reflection in the x-axis, since $a > 0$. Row 2 also has $h = -2$ and $k = -3$. Therefore it has a shift of left 2 and down 3 from the graph of $y = x^2$. It has a vertex at $(-2, -3)$ and has an axis of symmetry at $x = -2$.

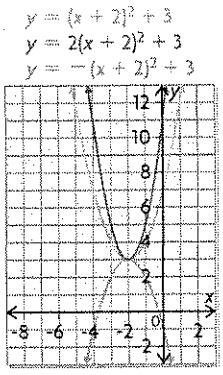
Row 3 has $a = -1$; so, it has no stretch/compression factor, but there is reflection in the x-axis. Row 3 also has $h = 1$ and $k = 4$. Therefore it has a shift of right 1, up 4 from the graph of $y = x^2$, has a vertex at $(1, 4)$, and has an axis of symmetry at $x = 1$.

Row 4 has $a = 0.8$; so, its compression factor is 0.8, and there is no reflection in the x-axis. Row 4 also has $h = 6$ and $k = 0$. Therefore it has a shift of right 6 from the graph of $y = x^2$, has a vertex at $(6, 0)$, and has an axis of symmetry at $x = 6$.

Row 5 has $a = 2$; so, its stretch factor is 2, and there is no reflection in the x-axis. Row 5 also has $h = 0$ and $k = -5$. Therefore it has a shift of down 5 from the graph of $y = x^2$, has a vertex at $(0, -5)$, and has an axis of symmetry at $x = 0$.

Quadratic Relation	Stretch/ Compression Factor	Reflection in the x-axis	Horizontal/ Vertical Translation	Vertex	Axis of Symmetry
$y = 3(x - 2)^2 - 5$	3	no	right 2, down 5	$(2, -5)$	$x = 2$
$y = 4(x + 2)^2 - 3$	4	no	left 2, down 3	$(-2, -3)$	$x = -2$
$y = -(x - 1)^2 + 4$	1	yes	right 1, up 4	$(1, 4)$	$x = 1$
$y = 0.8(x - 6)^2$	0.8	no	right 6	$(6, 0)$	$x = 6$
$y = 2x^2 - 5$	2	no	down 5	$(0, -5)$	$x = 0$

9. For example, the parabolas (1) $y = (x + 2)^2 + 3$, (2) $y = 2(x + 2)^2 + 3$, and (3) $y = -(x + 2)^2 + 3$ all have a vertex at $(-2, 3)$ because $h = -2$ and $k = 3$. Equation (1) has $a = 1$, so its graph looks like a standard parabola, only shifted. Equation (2) has $a = 2$, so its graph looks similar to that of equation (1), except that it is vertically stretched by a factor of 2. Equation (3) has $a = -1$, so its graph looks similar to that of equation (1), except that it opens down since $a < 0$. When sketched, the graphs look as follows:



10. a) In the equation for height, $a = -0.5(g - r)$. The smaller r is, the greater a will be in magnitude. The greater in magnitude a is, the narrower the parabola will be. Therefore, the graph of the bed sheet will be the narrowest parabola, the car tarp graph will be wider than that of the bed sheet, and the regular parachute graph will produce the widest parabola of all three.

Insert g and r to determine a.
Bed sheet:

$a = -0.5(g - r)$
$a = -0.5(9.8 - 0.6)$
$a = -0.5(9.2)$
$a = -4.6$

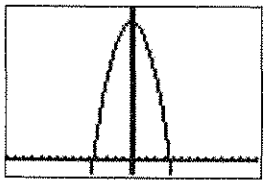
Car Tarp:

$a = -0.5(g - r)$
$a = -0.5(9.8 - 2.1)$
$a = -0.5(7.7)$
$a \doteq -3.9$

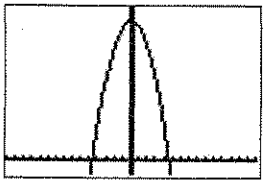
Parachute:

$a = -0.5(g - r)$
$a = -0.5(9.8 - 8.9)$
$a = -0.5(0.9)$
$a \doteq -0.5$

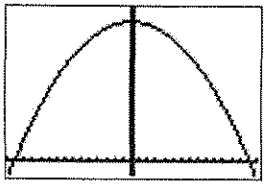
The value of a for each material reveals that the graph for the bed sheet has a stretch factor of 4.6, the graph for the car tarp has a lesser stretch factor of 3.9, and the graph for the parachute has a compression factor of 0.5. This confirms that the parabolas will widen from bed sheet to car tarp to parachute.
Graph each parabola to compare them.
Graph of bed sheet:



Graph of car tarp:



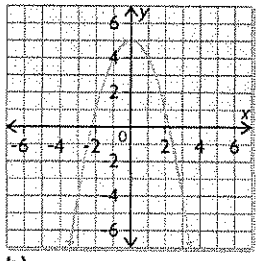
Graph of regular parachute:



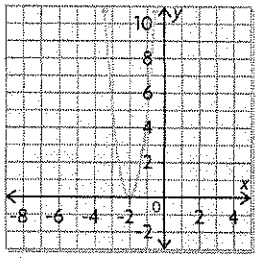
An object dropped from 100 m will hit the ground at about 4.5 s with a bed sheet, at about 5 s with a car tarp, and about 15 s with a regular parachute.
b) Yes, if the object dropped with the bed sheet is dropped from a much higher altitude than the object dropped from the regular parachute, or at a later time, then they could hit the ground at the same time. The graph for the bed sheet would be narrower than that for the regular parachute, but would also have a much higher vertex if dropped from a higher altitude. The positive zeros would be equal.

11. a) A reflection about the x-axis means $a < 0$. Since there is no stretch or compression factor, then $a = -1$. A translation of 5 units up means $k = 5$. The equation of the parabola is $y = -x^2 + 5$.
b) A vertical stretch by a factor of 5 means $a = 5$. A translation of 2 units left means that $h = -2$. The equation of the parabola is $y = 5(x + 2)^2$.
c) A vertical compression by a factor of $\frac{1}{5}$ means $a = \frac{1}{5}$. A translation of 6 units down means $k = -6$. The equation of the parabola is $y = \frac{1}{5}x^2 - 6$.
d) A reflection about the x-axis means $a < 0$. Since there is also a vertical stretch by a factor of 6, then $a = -6$. A translation of 3 units right and 4 units up means $h = 3$ and $k = 4$. The equation of the parabola is $y = -6(x - 3)^2 + 4$.

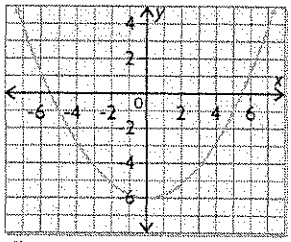
12. a)



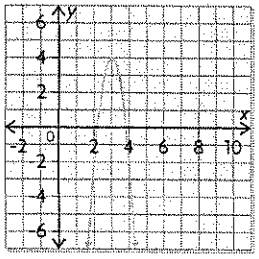
b)



c)



d)



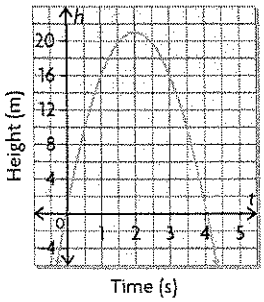
13. Equation c). The graph is of a parabola with a vertex at (3, 5); so, its equation must have $h = 3$ and $k = 5$. If the value of a in the equation was 1 or –1, the parabola would pass through (1, 1) and (5, 1). The graph is wider than a parabola passing through these points, so the graph is a result of a vertical compression of $y = x^2$. This rules out $y = -1(x - 3)^2 + 5$, equation b).

Thus, the equation is $y = -\frac{2}{3}(x - 3)^2 + 5$. This corresponds to equation c).

14. a) The sky diver has to have started his jump from the highest point in the parabola, the vertex. The equation has $h = 4$; so, the vertex has been shifted 4 units to the right. Thus, the sky diver delayed his jump by 4 s.

b) The equation has $k = 2500$; so, it has a vertex that has been shifted 2500 units up. The sky diver must then have jumped from a height of 2500 m, since he began his jump from the vertex of the parabola.

15. a) This equation has $a = -5$, $h = 2$, and $k = 21$. To sketch the graph, reflect the graph of $y = x^2$ about the x-axis and stretch the result vertically by a factor of 5. Then, translate the graph right 2 units and up 21 units. The final sketch should look like as follows:



b) The baseball reaches its maximum height at its vertex, which occurs at a height of 21 m since $k = 21$.

c) The baseball reaches its maximum height at its vertex, which occurs at $t = 2$ s since $h = 2$.

d) Inspecting the graph reveals the baseball reached a height of 10 m at the times of about 0.5 s and 3.5 s.

e) Inspecting the graph reveals the baseball hit the ground after about 4 s after it was hit.

16. For example, one way to perform this shift involves a translation. A translation right 5 and up 8 moves the point to the desired position: 3 + 5 = 8, and 9 + 8 = 17. Letting $h = 5$ and $k = 8$ performs this shift. The equation for the new parabola would then be $y = (x - 5)^2 + 8$. Another way to perform this shift involves reflecting about the x-axis and then applying a translation. After reflecting the parabola about the x-axis by letting $a = -1$, the point (3, 9) moves to (3, –9). Following up with a translation right 5 units and up 26 units moves the point to the desired position: 3 + 5 = 8, and –9 + 26 = 17. Letting $h = 5$ and $k = 26$ performs this translation. The equation for the new parabola would then be $y = -(x - 5)^2 + 26$.

Finally, this shift can be completed using a vertical stretch and a translation. Apply a stretch factor of $\frac{17}{9}$ using $a = \frac{17}{9}$. The points (3, 9) then shifts to $(3, \frac{17}{9} \times 9)$, or (3, 17). Following up with a translation right 5 units moves the point to the desired position: 3 + 5 = 8. Letting $h = 5$ and $k = 0$ performs this translation. The equation for the new parabola would then be $y = \frac{17}{9}(x - 5)^2$.

17. Expand this relation to write it in standard form.

$$\begin{aligned} y &= 2(x - 4)(x + 10) \\ y &= 2(x^2 - 4x + 10x - 40) \\ y &= 2(x^2 + 6x - 40) \\ y &= 2x^2 + 12x - 80 \end{aligned}$$

This relation is $y = 2x^2 + 12x - 80$ in standard form. To write the equation in vertex form, notice that the from the factored equation, the zeros are 4 and –10. By symmetry, the vertex falls exactly between these two points.

So the vertex occurs at $x = -3$. Inserting $x = -3$ into the equation gives:

$$\begin{aligned} y &= 2(x - 4)(x + 10) \\ y &= 2(-3 - 4)(-3 + 10) \\ y &= 2(-7)(7) \\ y &= -98 \end{aligned}$$

The vertex of this parabola occurs at (–3, –98). This means in vertex form, the relation has $h = -3$ and $k = -98$. Then, the equation in vertex form is $y = a(x + 3)^2 - 98$. Use one of the zeros to solve for a.

$$\begin{aligned} 0 &= a(4 + 3)^2 - 98 \\ 98 &= (7^2)a \\ 98 &= 49a \\ a &= 2 \end{aligned}$$

This relation is $y = 2(x + 3)^2 - 98$ in vertex form.

18. This quadratic relation has $a = -2$; so, it has been reflected in the x-axis and stretched by a factor of 2. Also, it has $h = -3$ and $k = 4$; so, it represents a translation left 3 units and up 4 units from $y = x^2$, putting its vertex at (–3, 4).

Translation: 3 units left, 4 units up	Reflection: reflected in the x-axis
$y = -2(x + 3)^2 + 4$	
Stretch/Compression: stretch by a factor of 2	Vertex: (–3, 4)

19. Let $y = 0$ to solve for a zero.

$$y = \left(x - \frac{k}{2}\right)^2 - \frac{(k-2)^2}{4}$$

$$0 = \left(x - \frac{k}{2}\right)^2 - \frac{(k-2)^2}{4}$$

$$\frac{(k-2)^2}{4} = \left(x - \frac{k}{2}\right)^2$$

$$\pm\sqrt{\frac{(k-2)^2}{4}} = \pm\sqrt{\left(x - \frac{k}{2}\right)^2}$$

To determine one zero, take the positive or negative roots only. Taking the positive roots,

$$\frac{k-2}{2} = x - \frac{k}{2}$$

$$\frac{k-2}{2} + \frac{k}{2} = x$$

$$\frac{2k-2}{2} = x$$

$$x = k - 1$$

To determine the other zero, take one positive and one negative root.

$$\frac{k-2}{2} = -\left(x - \frac{k}{2}\right)$$

$$\frac{k-2}{2} = -x + \frac{k}{2}$$

$$x = \frac{k}{2} - \frac{k-2}{2}$$

$$x = \frac{k-k+2}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

The zeros are 1 and $k - 1$.

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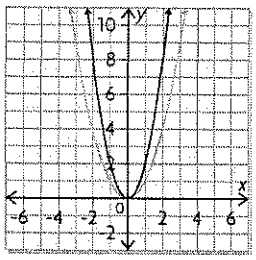
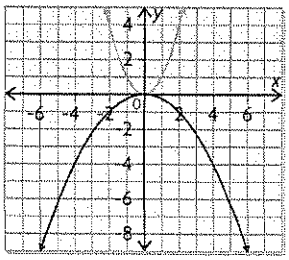
1. a)



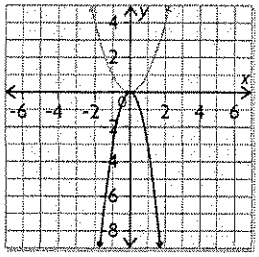
Transformations: The original graph was streched vertically by 2.

b)



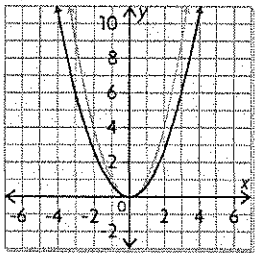
Transformation: The original graph was compressed vertically by a factor of 0.5 and was reflected about the x-axis.

c)



Transformation: The original graph was streched vertically by a factor of 3 and was reflected about the x-axis.

d)

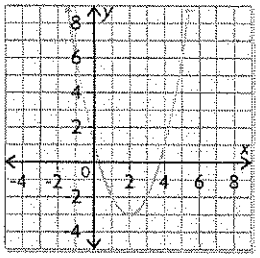


Transformation: The original graph was vertically compressed by a factor of $\frac{2}{3}$.

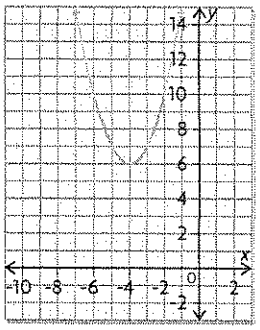
2. a) The black graph was vertically stretched by a factor of 4. Its equation is $y = 4x^2$.

b) The black graph was reflected about the x-axis. Its equation is $y = -x^2$.

3. a) Two units to the right means $h = 2$ and down three units means $k = -3$. The equation is $y = (x - 2)^2 - 3$.



b) Four units to the left means $h = -4$ and up six units means $k = 6$. The equation would be $y = (x + 4)^2 + 6$.



4. All of the parabolas have the same shape, so I can keep $a = 1$ for all of them. The vertices are (–3, –3), (–2, –2), (–1, –1), (0, 0), (1, 1), (2, 2) and (3, 3). So the equations would be:
$y = (x + 3)^2 - 3$, $y = (x + 2)^2 - 2$, $y = (x + 1)^2 - 1$, $y = x^2$, $y = (x - 1)^2 + 1$, $y = (x - 2)^2 + 2$, $y = (x - 3)^2 + 3$.

5. a) Start with a vertical stretch by a factor of 3 then a reflection in the x-axis and a translation 1 unit to the right.

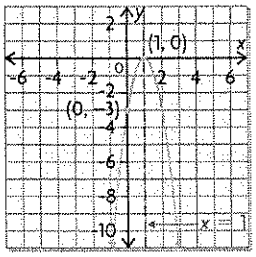
b) Apply a vertical compression by a factor of 0.5 then translate the graph 3 units to the left and 8 units down.

c) Vertically stretch the graph by a factor of 4 then translate it 2 units to the right and 5 units down.

d) Vertically compress the graph by a factor of $\frac{2}{3}$ and translate it 1 unit down.

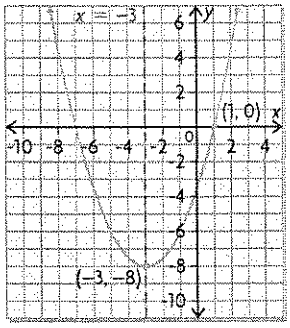
6. Answers may vary, e.g.,

a)



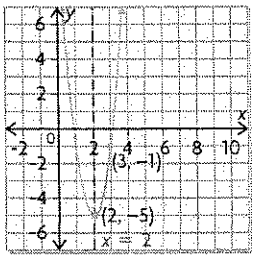
The vertex is (1, 0). There is a vertical stretch by a factor of –3. The point (0, 0) is transformed into (0, –3).

b)



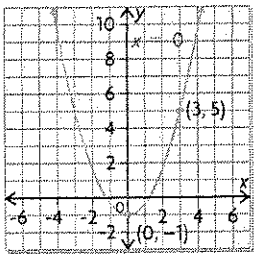
The vertex is (–3, –8). There is a vertical compression by a factor of 0.5. The point (1, 1) is transformed into (1, 0).

c)



The vertex is (2, –5). There is a vertical stretch by a factor of 4. The point (1, 1) is transformed into (1, –1).

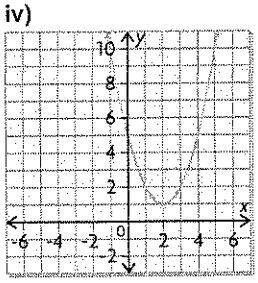
d)



The vertex is (0, –1). There is a vertical compression by a factor of $\frac{2}{3}$. The point (1, 1) is transformed into $\left(1, \frac{2}{3}\right)$.

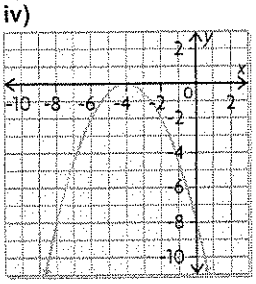
7. a) i) Stretch $a = 1$, translation 2 units right and 1 unit up. **ii)** No reflection, since a is positive. **iii)** The vertex is (2, 1), so the axis of symmetry is $x = 2$.

iv)



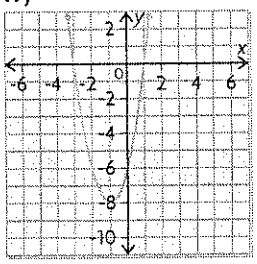
b) i) There is a compression factor 0.5, and a translation 4 units left. **ii)** There is a reflection since a is negative. **iii)** The vertex is (–4, 0), so the equation of the axis of symmetry is $x = -4$.

iv)



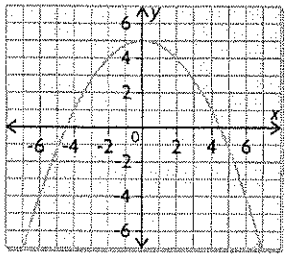
c) i) There is a stretch factor of 2, a translation 1 unit left, and 8 units down. **ii)** No reflection since a is positive. **iii)** The vertex is (–1, –8) and the equation of the axis of symmetry is $x = -1$.

iv)



d) i) There is a compression factor of 0.25 and a translation 5 units up. **ii)** There is a reflection since a is negative. **iii)** The vertex is (0, 5), so the equation of the axis of symmetry is $x = 0$.

iv)



8. The parabola is either in the top two quadrants or the bottom two. For the top quadrants, $a > 0$, $k > 0$, and h can take any value. E.g., $y = 3x^2 + 2$. For bottom quadrants, $a < 0$, $k < 0$, h can take any value. E.g., $y = -3x^2 - 2$.

Lesson 5.4: Quadratic Models Using Vertex Form, page 275

1. a) The vertex is (0, –8), so the graph must be iii).
b) The vertex is (–3, 0), so the graph must be iv).
c) The vertex is (4, 8), so the graph must be i).
d) The vertex is (3, –8), so the graph must be ii).

2. a) The equation that represents this family of parabolas is $y = a(x - 4)^2 - 12$, $a \neq 0$.

b) To determine the a-value, evaluate the equation at (13, 15).

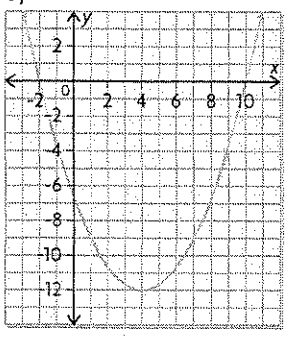
$$y = a(x-4)^2 - 12$$
$$15 = a(13-4)^2 - 12$$
$$15 = a(9)^2 - 12$$
$$27 = 81a$$
$$a = \frac{1}{3}$$

c) The equation of the relation in part b) is

$$y = \frac{1}{3}(x-4)^2 - 12.$$

d) Apply a vertical compression by a factor of $\frac{1}{3}$, a translation 4 units right and 12 units down.

e)



3. a) The vertex is (0, 0) and a point of the graph is (2, 1). The equation in vertex form is $y = a(x + 0)^2 + 0$. Solve for a using (2, 1).

$$y = ax^2$$
$$1 = a(2)^2$$
$$1 = 4a$$
$$a = 0.25$$

The equation of the graph is $y = 0.25x^2$.

b) The vertex is (–1, 0) and a point of the graph is (–2, 2). The equation in vertex form is $y = a(x + 1)^2 + 0$. Solve for a using (–2, 2).

$$y = a(x + 1)^2$$
$$2 = a(-2 + 1)^2$$
$$2 = a$$

The equation of the graph is $y = 2(x + 1)^2$.

c) The vertex is (0, 4) and a point of the graph is (2, 0). The equation in vertex form is $y = a(x + 0)^2 + 4$. Solve for a using (2, 0).

$$0 = a(2 + 0)^2 + 4$$
$$0 = 4a + 4$$
$$a = -1$$

The equation of the graph is $y = -x^2 + 4$.

d) The vertex is (1, 2) and a point of the graph is (–1, –2). The equation in vertex form is $y = a(x - 1)^2 + 2$. Solve for a using (–1, –2).

$$-2 = a(-1 - 1)^2 + 2$$
$$-2 = 4a + 2$$
$$-1 = a$$

The equation of the graph is $y = -(x - 1)^2 + 2$.

e) The vertex is (2, 4) and a point of the graph is (3, 1). The equation in vertex form is $y = a(x - 2)^2 + 4$. Solve for a using (3, 1).

$$1 = a(3 - 2)^2 + 4$$
$$a = -3$$

The equation of the graph is $y = -3(x - 2)^2 + 4$.

f) The vertex is (–2, –3) and a point of the graph is (–3, 2). The equation in vertex form is $y = a(x + 2)^2 - 3$. Solve for a using (–3, 2).

$$2 = a(-3 + 2)^2 - 3$$
$$2 = a - 3$$
$$5 = a$$

The equation of the graph is $y = 5(x + 2)^2 - 3$.

4. a) Since there is a vertical stretch, a changes; the new equation is $y = 4x^2$.

b) Since there is a translation left, the x-value of the vertex changes; the new equation is $y = (x + 3)^2$.

c) A reflection about the x-axis gives a negative a-value and a translation up causes a change in the y-value of the vertex. The new equation is $y = -x^2 + 2$.

d) Since there is a vertical compression a, changes; the new equation is $y = \frac{1}{2}x^2$.

e) The shift to the right changes the x-value of the vertex and the shift down changes the y-value. The new equation is $y = (x - 5)^2 - 4$.

f) The reflection is represented by a negative *a*-value, the stretch is also represented by a change to the *a*-value. Finally, the shift left is represented by a change in the *x*-value of the vertex. The new graph is $y = -2(x + 1)^2$.

5. a) Upward-opening means that *a* is positive and the same shape means $a = 1$. The equation is $y = x^2 + 4$.

b) Downward-opening means *a* is negative and the same shape means $a = -1$. The equation is $y = -(x - 5)^2$.

c) Answers may vary, e.g., upward-opening means *a* is positive and a narrower shape means $a > 1$. A possible equation is $y = 2(x - 2)^2 - 3$.

d) Answers may vary, e.g., downward-opening means *a* is negative and a wider shape means $-1 < a < 0$. A possible equation is $y = -0.5(x + 3)^2 + 5$.

e) Answers may vary, e.g., since the equation of the axis of symmetry is $x = 4$ then the vertex must be $(4, y)$, but I also know the parabola crosses the *x*-axis twice, so the *y*-value in the vertex must be negative. In addition, upward-opening means *a* is positive, and a narrower shape means $a > 1$. A possible equation is $y = 2(x - 4)^2 - 8$.

f) Answers may vary, e.g., since there are no zeros then the parabola must open downward, because it has to stay on the same side of the *x*-axis at the vertex. A wider shape means $-1 < a < 0$. A possible equation is $y = -0.5(x - 3)^2 - 4$.

6. a) Insert *h* and *k*: $y = a(x + 2)^2 + 3$

Evaluate the equation at $(-4, 1)$ to determine *a*.

$1 = a(-4 + 2)^2 + 3$

$1 = 4a + 3$

$a = -0.5$

The equation is $y = -0.5(x + 2)^2 + 3$.

b) Insert *h* and *k*: $y = a(x + 1)^2 - 1$

Evaluate the equation at $(0, 1)$ to determine *a*.

$1 = a(0 + 1)^2 - 1$

$1 = a - 1$

$a = 2$

The equation is $y = 2(x + 1)^2 - 1$.

c) Insert *h* and *k*: $y = a(x + 2)^2 - 3$

Evaluate the equation at $(-5, 6)$ to determine *a*.

$6 = a(-5 + 2)^2 - 3$

$6 = 9a - 3$

$a = 1$

The equation is $y = (x + 2)^2 - 3$.

d) Insert *h* and *k*: $y = a(x + 2)^2 + 5$

Evaluate the equation at $(1, -4)$ to determine *a*.

$-4 = a(1 + 2)^2 + 5$

$-4 = 9a + 5$

$a = -1$

The equation is $y = -(x + 2)^2 + 5$.

7. a) The equation of the axis of symmetry is $x = 5$ since the *y*-values on either side of it are equal. The vertex is $(5, 3)$. To determine *a*, insert the vertex and a point into the equation in vertex form:

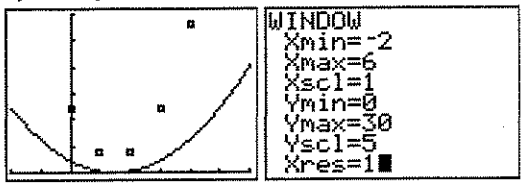
$y = a(x - 5)^2 + 3$

$-1 = a(6 - 5)^2 + 3$

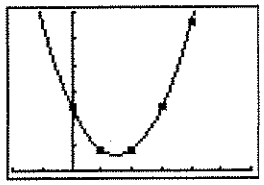
$-4 = a$

The equation is $y = -4(x - 5)^2 + 3$.

b) The equation of the axis of symmetry is $x = 1.5$ since the *y*-values on either side of it are equal. I entered the data into a graphing calculator and substituted the axis of symmetry into the vertex form: $y = (x - 1.5)^2$.

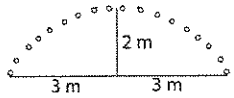


I adjusted the stretch factor and height; I found that $a = 4$ and $k = 3$ gave the right fit.



The equation is $y = 4(x - 1.5)^2 + 3$.

8. Answers may vary, e.g.,



Assume that child is standing at $(0, 0)$ and that when the ball cleared the fence it was at its highest point $(3, 2)$. When the ball lands it be at position $(6, 0)$. Use factored form and evaluate it at $(3, 2)$ to determine the value of *a*.

$y = a(x - 6)(x - 0)$

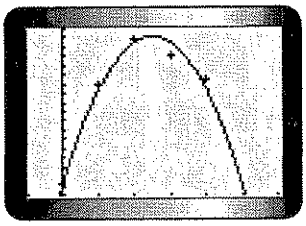
$2 = a(3 - 6)(3 - 0)$

$2 = -9a$

$a = -\frac{2}{9}$

Insert *a* and the vertex $(3, 2)$ into the vertex form of the equation. The equation is $y = -\frac{2}{9}(x - 3)^2 + 2$.

9. a)



b) Answers may vary, e.g., I estimate the vertex is about $(2.5, 4625)$. In the graph, the point $(0, 1446)$ appears to be on the parabola, so I will use this point to determine the *a*-value.

$y = a(x - 2.5)^2 + 4625$

$1446 = a(0 - 2.5)^2 + 4625$

$1446 = 6.25a + 4625$

$-3179 = 6.25a$

$a = -509$

The equation for this model is $y = -509(x - 2.5)^2 + 4625$.

c) To determine the number of DVDs sold in 2010, solve the equation when $x = 8$.

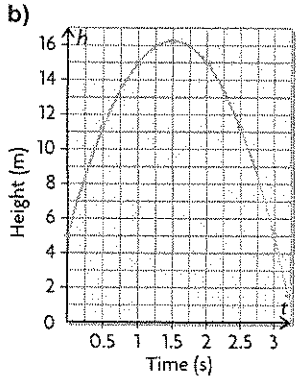
$$y = -509(8 - 2.5)^2 + 4625$$
$$y = -509(30.25) + 4625$$
$$y = -10772.25$$

Since there cannot be a negative number of DVDs then by 2010 there will not be any DVDs sold at all.

d) Answers may vary, e.g., The equation I get using regression is $y = -484x^2 + 2440x + 1553$. If I expand my equation from part b), I get $y = -509x^2 + 2545x + 1443.75$. My estimate was reasonably accurate.

10. a) The data appears to be quadratic since the height values increase then decrease.

b)



c) Answers may vary, e.g., I estimate the vertex is about (1.5, 16.25).

d) Since the first point, (0, 5), lies on the graph, I will use it to determine the a-value.

$$5 = a(0 - 1.5)^2 + 16.25$$
$$5 = 2.25a + 16.25$$
$$-11.25 = 2.25a$$
$$a = -5$$

The equation is $h = -5(t - 1.5)^2 + 16.25$.

e) After 2.75 s:

$$h = -5(2.75 - 1.5)^2 + 16.25$$
$$h = -7.8125 + 16.25$$
$$h = 8.4375$$

After 1.25 s:

$$h = -5(1.25 - 1.5)^2 + 16.25$$
$$h = -0.3125 + 16.25$$
$$h = 15.9375$$

The height of the ball at 2.75 s is 8.4735 m, and after 1.25 s it is 15.9375 m.

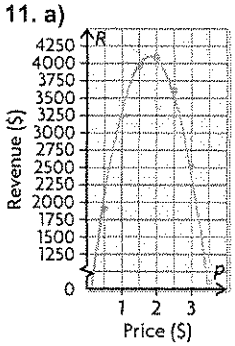
f) To see how effective my model is for values greater than 3.5 s, substitute such a value in my equation:

$$h = -5(3.5 - 1.5)^2 + 16.25$$
$$h = -5(4) + 16.25$$
$$h = -3.75$$

My model is not effective: the height is negative, which would mean the ball is under ground level.

g) The equation from the regression is $h = -5t^2 + 15t + 5$. If I write the equation from part b) in standard form, I get $h = -5t^2 + 15t + 5$. My estimate is highly accurate.

11. a)



b) Answers may vary, e.g., I estimate the vertex is about (1.8, 4100). The point (2.5, 3512) appears to be on the graph, so use it to determine the a-value.

$$R = a(p - 1.8)^2 + 4100$$
$$3512 = a(2.5 - 1.8)^2 + 4100$$
$$-588 = 0.49a$$
$$a = -1200$$

The equation in vertex form is:
$R = -1200(p - 1.8)^2 + 4100$

c) Substitute 2.25 in my equation to determine the revenue:

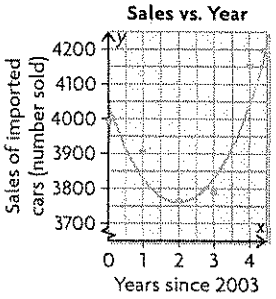
$$R = -1200(2.25 - 1.8)^2 + 4100$$
$$= 3857$$

Answers may vary, e.g., if an ice cream cone were $2.25, then revenue would be about $4000 to the nearest thousand.

d) Answers may vary, e.g., since the vertex is at (1.8, 4100), then to maximize revenue, an ice cream cone should cost $1.80.

e) The equation for the regression is $R = -1200p^2 + 4440p$. The standard form of the relation from part b) is $R = -1200p^2 + 4320p + 212$. This estimate is reasonably accurate.

12. a) Answers may vary, e.g.,



b) Answers may vary, e.g., the vertex is about (2, 3760) and point (4, 4040) appears to be on the curve, so I will use it to determine a.

$$4040 = a(4 - 2)^2 + 3760$$
$$4a = 280$$
$$a = 70$$

The equation of my estimate is $y = 70(x - 2)^2 + 3760$.

c) The year 2008 is 5 years from 2003, so $x = 5$:

$$y = 70(5 - 2)^2 + 3760$$
$$y = 4390$$

There were about 4390 cars sold.

d) The year 2006 is 3 years from 2003, so $x = 3$:

$y = 70(3-2)^2 + 3760$

$y = 3830$

They sold about 3830 cars. This is reasonably accurate since it is about 1.5% higher than the actual value.

e) The equation for the regression is $y = 77x^2 - 288x + 4036$. My model in part b), in standard form, is $y = 70x^2 - 280x + 4040$. This model is reasonably accurate.

13. The top of the first tower and the suspension cord will be at point (0, 50) since it is at the starting point and 50 m above the bridge. The vertex of the suspension cable will be at (758, 2) since it is halfway across the bridge and 2m above the bridge. Since I have the vertex, I will use the vertex form to represent this relation. Let h represent the height of the large cables from the road and x represent the horizontal distance from the first tower. So far I have $h = a(x - 758)^2 + 2$. To determine a, substitute point (0, 50) into the equation.

$50 = a(0-758)^2 + 2$

$48 = 574564a$

$a = 0.000\,083\,5$

The equation for the large cables is $h = 0.000\,083\,5(x - 758)^2 + 2$.

14. Strategy 1: The vertex is (20, 2000); substitute (40, 0) in $h = a(t-20)^2 + 2000$ to determine value of a.
Strategy 2: The two zeros are (0, 0) and (40, 0); substitute the vertex (20, 2000) in $h = at(t-40)$ to determine value of a.

15. I know the maximum profit was \$1600 when the price was \$75. This is the vertex, (75, 1600). I also know point (50, 1225) is on the parabola. Use the vertex and the point to solve for a.

$1225 = a(50-75)^2 + 1600$

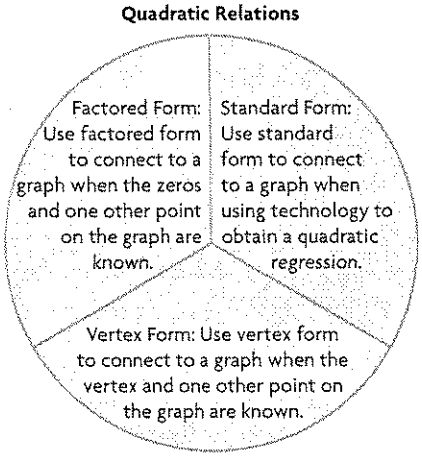
$1225 = 625a + 1600$

$-375 = 625a$

$a = -0.6$

The equation is $p = -0.6(d-75)^2 + 1600$.

16. Answers may vary, e.g.,


Quadratic Relations
Factored Form: Use factored form to connect to a graph when the zeros and one other point on the graph are known.
Standard Form: Use standard form to connect to a graph when using technology to obtain a quadratic regression.
Vertex Form: Use vertex form to connect to a graph when the vertex and one other point on the graph are known.

17. a) A translation 4 units to the right causes a change in h: the equation is $y = 2(x-1)^2 - 1$.

b) A reflection across the x-axis causes a change in a's sign, so the equation is $y = -2(x+3)^2 - 1$.

c) First apply the reflection across the x-axis, which causes a change a's sign. The vertex changes to (–3, 1). Then apply the translation 5 units down. The equation is $y = -2(x+3)^2 - 4$.

d) A stretch causes a change in a: $a = 2 \times 6 = 12$. The equation is $y = 12(x+3)^2 - 1$.

e) A compression causes a change in a: $a = 2 \times 0.25 = 0.5$. A reflection in the y-axis means the x-values of the parabola change. Since the equation is in vertex form, I just need to recalculate x-value of the vertex. Since $x = -3$, the reflection results in $x = 3$. The equation is $y = 0.5(x-3)^2 - 6$.

18. I know $a = 3$ and the vertex is (–1, 4), so the vertex form of the equation is $y = 3(x+1)^2 + 4$. Expand to write it in standard form.

$y = 3(x+1)^2 + 4$

$y = 3(x^2 + 2x + 1) + 4$

$y = 3x^2 + 6x + 7$

Therefore, $b = 6$ and $c = 7$.

19. Isolate x:

$$a(x-h)^2 + k = 0$$
$$a(x-h)^2 = -k$$
$$(x-h)^2 = -\frac{k}{a}$$
$$x - h = \pm\sqrt{-\frac{k}{a}}$$
$$x = h \pm \sqrt{-\frac{k}{a}}$$

Lesson 5.5: Solving Problems Using Quadratic Relations, page 285

1. a) $y = 2(x-0)^2 + 3$

$y = 2x^2 + 3$

b) $y = -3(x-2)^2 + 0$

$y = -3(x-2)^2$

c) $y = -(x-3)^2 - 2$

d) $y = 0.5(x+3.5)^2 + 18.3$

2. a) The parabola has a minimum, since a is positive. The minimum is 3.

b) The parabola has a maximum, since a is negative. The maximum is 0.

c) The parabola has a maximum, since a is negative. The maximum is –2.

d) The parabola has a minimum, since a is positive. The minimum is 18.3.

3. a) The vertex of the graph is (0, 0), so it is of the form $y = ax^2$, and because the graph opens downward, a is negative.
Point (–4, –1) is a on the graph. Use this point to determine a.

$y = ax^2$

$-1 = a(-4)^2$

$-1 = 16a$

$a = -0.0625$

An equation of the parabola is $y = -0.0625x^2$.

b) The value of a is the same in each of these equations, since the parabolas are congruent.

4. a) $y = a(x - 0)^2 + 3$ evaluated at (2, –5):

$-5 = a(2 - 0)^2 + 3$

$-5 = 4a + 3$

$-8 = 4a$

$a = -2$

The equation is $y = -2x^2 + 3$.

b) $y = a(x - 2)^2 + 0$ evaluated at (5, 9):

$9 = a(5 - 2)^2$

$9 = 9a$

$a = 1$

The equation is $y = (x - 2)^2$.

c) $y = a(x + 3)^2 + 2$ evaluated at (– 1, 14):

$14 = a(-1 + 3)^2 + 2$

$14 = 4a + 2$

$12 = 4a$

$a = 3$

The equation is $y = 3(x + 3)^2 + 2$.

d) $y = a(x - 5)^2 - 3$ evaluated at (1, –8):

$-8 = a(1 - 5)^2 - 3$

$-8 = 16a - 3$

$-5 = 16a$

$a = \frac{-5}{16}$

The equation is $y = -\frac{5}{16}(x - 5)^2 - 3$.

5. a) The vertex of the parabola is (–1, –4). This gives the equation $y = a(x + 1)^2 - 4$. To determine the a-value, evaluate at (1, 0).

$0 = a(1 + 1)^2 - 4$

$0 = 4a - 4$

$a = 1$

The equation is $y = (x + 1)^2 - 4$.

b) The vertex of the parabola is (4, –2). This gives the equation $y = a(x - 4)^2 - 2$. To determine the a-value, evaluate at (6, 6).

$6 = a(6 - 4)^2 - 2$

$6 = 4a - 2$

$a = 2$

The equation is $y = 2(x - 4)^2 - 2$.

c) The vertex of the parabola is (4, 4). This gives the equation $y = a(x - 4)^2 + 4$. To determine the a-value, evaluate at (2, 0).

$0 = a(2 - 4)^2 + 4$

$0 = 4a + 4$

$a = -1$

The equation is $y = -(x - 4)^2 + 4$.

d) The vertex of the parabola is (0, 4). This gives the equation $y = a(x - 0)^2 + 4$. To determine the a-value, evaluate at (2, 2).

$2 = a(2 - 0)^2 + 4$

$2 = 4a + 4$

$a = -\frac{1}{2}$

The equation is $y = -\frac{1}{2}x^2 + 4$.

6. a) From the graph, the x-intercepts are –3 and 1. Also, from the vertex form, $a = 1$. The factored form of this equation is $y = (x - 1)(x + 3)$. To derive the standard form, expand.

$y = (x - 1)(x + 3)$

$y = x^2 - x + 3x - 3$

$y = x^2 + 2x - 3$

The equation in standard form is $y = x^2 + 2x - 3$.

b) From the graph, the x-intercepts are 5 and 3. Also, from the vertex form, $a = 2$. The factored form of this equation is $y = 2(x - 5)(x - 3)$. To derive the standard form, expand.

$y = 2(x - 5)(x - 3)$

$y = 2(x^2 - 5x - 3x + 15)$

$y = 2x^2 - 16x + 30$

The equation in standard form is $y = 2x^2 - 16x + 30$.

c) From the graph, the x-intercepts are 2 and 6. Also, from the vertex form, $a = -1$. The factored form of this equation is $y = -(x - 2)(x - 6)$. To derive the standard form, expand.

$y = -(x - 2)(x - 6)$

$y = -(x^2 - 2x - 6x + 12)$

$y = -x^2 + 8x - 12$

The equation in standard form is $y = -x^2 + 8x - 12$.

d) It is not clear from the graph what the x-intercepts are, so solve the equation from question 5 for $y = 0$.

$y = -\frac{1}{2}x^2 + 4$

$0 = -\frac{1}{2}x^2 + 4$

$\frac{1}{2}x^2 = 4$

$x^2 = 8$

$x^2 - 8 = 0$

Also, from vertex form, $a = -\frac{1}{2}$. The factored form of this equation is $y = -\frac{1}{2}(x^2 - 8)$. The standard form of this equation is the same as the vertex form, $y = -\frac{1}{2}x^2 + 4$.

7. Since I was given the x-intercepts in the question, I will begin working with the factored form. By filling in what I know, I get $y = a(x - 8)(x + 2)$. To determine a, evaluate the equation at (0, 8).

$8 = a(0 - 8)(0 + 2)$

$8 = a(-8)(2)$

$8 = -16a$

$a = -0.5$

The equation in factored form is $y = -0.5(x - 8)(x + 2)$. Since 8 and –2 are equidistant from the axis of symmetry, I can determine their midpoint: 3. To determine the vertex, evaluate the equation when $x = 3$.

$y = -0.5(3 - 8)(3 + 2)$

$y = -0.5(-5)(5)$

$y = 12.5$

The equation in vertex form is $y = -0.5(x - 3)^2 + 12.5$.

8. The minimum value of the equation is represented by the y-value within the vertex. Translating the parabola 5 units to the right will add 5 to the x-value and translating the parabola down 3 units will subtract 3 for the y-value. Since the vertex was (–4, –7) the new vertex will be (1, –10). Therefore the new minimum value is –10. The a-value does not change since a translation does not affect the shape of the parabola. The equation in vertex form is $y = 2(x - 1)^2 - 10$.

9. a) To write the equation in standard form, expand the vertex form:

$y = (x - 4)^2 - 1$
$y = x^2 - 8x + 16 - 1$
$y = x^2 - 8x + 15$

Factor the standard form:

$y = x^2 - 8x + 15$
$y = (x - 5)(x - 3)$

The equation is $y = x^2 - 8x + 15$ in standard form and $y = (x - 5)(x - 3)$ in factored form.

b) To write the equation in standard form, expand the vertex form:

$y = 2(x + 1)^2 - 18$
$y = 2(x^2 + 2x + 1) - 18$
$y = 2x^2 + 4x + 2 - 18$
$y = 2x^2 + 4x - 16$

Factor the standard form:

$y = 2x^2 + 4x - 16$
$y = 2(x^2 + 2x - 8)$
$y = 2(x + 4)(x - 2)$

The equation is $y = 2x^2 + 4x - 16$ in standard form and $y = 2(x + 4)(x - 2)$ in factored form.

c) To write the equation in standard form, expand the vertex form:

$y = -(x + 5)^2 + 1$
$y = -(x^2 + 10x + 25) + 1$
$y = -x^2 - 10x - 25 + 1$
$y = -x^2 - 10x - 24$

Factor the standard form:

$y = -x^2 - 10x - 24$
$y = -(x^2 + 10x + 24)$
$y = -(x + 4)(x + 6)$

The equation is $y = -x^2 - 10x - 24$ in standard form and $y = -(x + 4)(x + 6)$ in factored form.

d) To write the equation in standard form, expand the vertex form:

$y = -3(x + 3)^2 + 75$
$y = -3(x^2 + 6x + 9) + 75$
$y = -3x^2 - 18x - 27 + 75$
$y = -3x^2 - 18x + 48$

Factor the standard form:

$y = -3x^2 - 18x + 48$
$y = -3(x^2 + 6x - 16)$
$y = -3(x + 8)(x - 2)$

The equation is $y = -3x^2 - 18x + 48$ in standard form and $y = -3(x + 8)(x - 2)$ in factored form.

10.a) Factor the equation to get the factored form:

$y = 2x^2 - 12x$
$y = 2x(x - 6)$

From the standard form, $a = 2$. I can determine the axis of symmetry, $x = \frac{0 + 6}{2} = 3$.

Now I can use the axis of symmetry to determine the vertex:

$y = 2(3)^2 - 12(3)$
$y = 2(9) - 36 = -18$

The vertex is (3, – 18). The equation in factored form is $y = 2x(x - 6)$ and in vertex form is $y = 2(x - 3)^2 - 18$.

b) Factor the equation to get the factored form:

$y = -2x^2 + 24x - 64$
$y = -2(x^2 - 12x + 32)$
$y = -2(x - 8)(x - 4)$

From the standard form, $a = -2$. I can determine the axis of symmetry, $x = \frac{6 + 6}{2} = 6$. Now I can use the axis of symmetry to determine the vertex:

$y = -2(6)^2 + 24(6) - 64$
$y = -72 + 144 - 64 = 8$

The vertex is (6, 8). The equation in factored form is $y = -2(x - 8)(x - 4)$ and in vertex form, $y = -2(x - 6)^2 + 8$.

c) Factor the equation to get the factored form:

$y = 2x^2 - x - 6$
$y = 2x^2 - 4x + 3x - 6$
$y = (2x + 3)(x - 2)$

From the standard form, $a = 2$. I can determine the axis of symmetry, $x = \frac{-1.5 + 2}{2} = 0.25$. Now I can use the axis of symmetry to determine the vertex:

$y = 2(0.25)^2 - (0.25) - 6$
$y = 0.125 - 0.25 - 6 = -6.125$

The vertex is (0.25, –6.125). The equation in factored form is $y = (2x + 3)(x - 2)$ and in vertex form is $y = 2(x - 0.25)^2 - 6.125$.

d) Factor the equation to get the factored form:

$y = 4x^2 + 20x + 25$
$y = 4x^2 + 10x + 10x + 25$
$y = 2x(2x + 5) + 5(2x + 5)$
$y = (2x + 5)^2$

From the standard form, $a = 4$. The axis of symmetry is at the zero: $x = -\frac{5}{2} = -2.5$. Notice that when $x = -2.5$, $y = 0$. Therefore, the equation in standard form is $y = (2x + 5)^2$ and in vertex form is $y = 4(x + 2.5)^2$.

11. Let x be the number of \$0.50 increases. The cover charge after x increases will be $5.00 + 0.50x$. The number of people after x increases is expressed by $300 - 30x$. To determine the total revenue, I will multiply the number of people by the price each of them will pay:
$R = (5.00 + 0.50x)(300 - 30x)$
To determine the maximum revenue, determine the vertex of the parabola. Use the x-intercepts to determine the axis of symmetry. The x-intercepts are –10 and 10, so the equation of the axis of symmetry is $x = 0$. The fare that will generate the highest revenue is $5.00 + 0.50(0) = \$5.00$. The cover charge should remain \$5.00 to have maximum profits.

12. Determine the translated parabola's equation. Since it will have the same a-value and I already know the x-intercepts, I will use the factored form.
$y = -2(x - 2)(x + 4)$

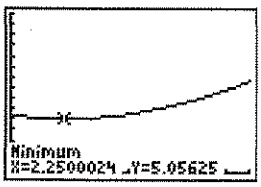
The axis of symmetry is $\frac{-4 + 2}{2} = -1$.

To determine the new vertex, evaluate the equation at $x = -1$.

$y = -2(x - 2)(x + 4)$
$y = -2(-1 - 2)(-1 + 4)$
$y = -2(-3)(3)$
$y = 18$

Now use the coordinates of both vertices to determine the translations. Moving from (–5, 8) to (–1, 18) is a translation 4 units right and 10 units up.

13. a) Because the equation is difficult to factor, use graphing technology or a table of values to determine the minimum.



Ticket prices were lowest about two years after 1995, in 1997.

b) The year 1998 is three years from 1995, so $t = 3$:

$C = 0.06(3)^2 - 0.27(3) + 5.36$
$= 0.54 - 0.81 + 5.36$
$= 5.09$

The average ticket price in 1998 was $5.09.

c) The year 2010 is 15 years from 1995, so $t = 15$:

$C = 0.06(15)^2 - 0.27(15) + 5.36$
$= 13.5 - 4.05 + 5.36$
$= 5.09$

The average ticket price in 2010 is estimated to be $14.81.

d) From the standard form, $a = 0.06$. Inserting the vertex from part a), the equation in vertex form is:
$C = 0.06(t - 2.25)^2 + 5.056\,25$.

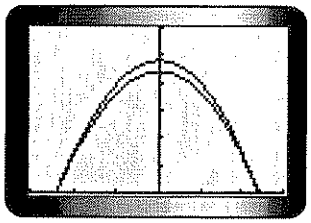
14. Answers may vary, e.g., since the height must be between 22 m and 26 m, I will choose 22 m and 24 m. The total distance at the base of the arch is 24 m. The arch will have x-intercepts of 12 and –12 which I got by dividing 24 by 2 and moving 12 units away from the origin to either side. Without taking the height of the bridge into account, the factored form is $y = a(x - 12)(x + 12)$. When the height of the bridge is 22 m, the maximum point is (0, 22). Find a for this height.

$y = a(x-12)(x+12)$
$22 = a(0-12)(0+12)$
$22 = -144a$
$a = -\frac{11}{72}$

Find the value of a if the height of the bridge is 24 m. When the height of the bridge is 24 m, the maximum point is (0, 24).

$y = a(x-12)(x+12)$
$24 = a(0-12)(0+12)$
$24 = -144a$
$a = -\frac{1}{6}$

Two of the possible equations for the bridge are $y = -\frac{11}{72}x^2 + 22$ and $y = -\frac{1}{6}x^2 + 24$.



15. a) From the factored form, I can determine the x-intercepts and the axis of symmetry. The x-intercepts are 15 and –11. The equation of the axis of symmetry is $x = \frac{-11+15}{2} = 2$. To determine the vertex, I evaluate the equation when $x = 2$.

$P = 20(15 - x)(11 + x)$
$P = 20(15 - 2)(11 + 2)$
$P = 20(13)(13)$
$P = 3380$

The vertex is (2, 3380). If I expanded the factored form, then the x^2 term would be –20, so $a = -20$. The equation in vertex form is $P = -20(x - 2)^2 + 3380$.

b) The number of $1 increases and the maximum profit can be determined from the vertex. The new ticket price is 11 + 2 = 13, and it gives a maximum profit of $3380. To determine the number of tickets sold, I will divide the profit by the ticket price: 3380 ÷ 13 = 260. There were 260 tickets sold.

16. Since the highest point of the arch is 30 m, this is the vertex, and I will assume it is perfectly centred, giving the vertex the following coordinates: (0, 30). If the bridge is 50 m wide, there must be 25 m on each side of the axis of symmetry. This would make the x-intercepts (25, 0) and (–25, 0). Now determine the equation for the arch by inserting the zeros and a point (the vertex) into the factored form:

$y = a(x-25)(x+25)$
$30 = a(-25)(25)$
$30 = -625a$
$a = -\frac{6}{125}$

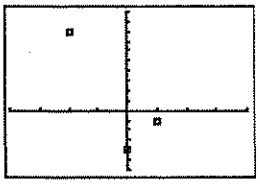
The equation is $y = -0.048(x - 25)(x + 25)$.

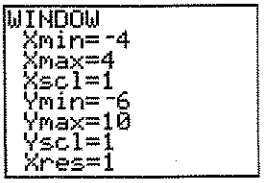
To determine the height of the bridge 8 m from the axis of symmetry, solve for $x = 8$.

$y = -0.048(x - 25)(x + 25)$
$y = -0.048(8 - 25)(8 + 25)$
$y = -0.048(-17)(33)$
$y = 26.928$

No, the boat cannot pass since clearance 8 m from axis of symmetry is only 26.928 m.

17. I know that a parabola is symmetrical, so I will start by determining the equation of the axis of symmetry. I notice that the y values go from 8 down to –4 and back up to –1. Therefore, the axis of symmetry must be between the x-values corresponding to 8 and –1: –2 and 1. To help me visualize the parabola, I plot the points.





If the axis of symmetry was at –2, then the points (0, –4) and (1, –1) would have to exist on the left side, but then the parabola's shape wouldn't be a U. I notice that the parabola would also not be a U if the axis of symmetry were at $x = -1$ or $x = 1$. It seems that at $x = 0$, the parabola is symmetrical. This makes the vertex (0, –4). To verify, I insert the zero into the vertex form of the equation, and substitute another point into the equation to solve for a. I determine that the equation is $y = 3x^2 - 4$. I verify that all the points satisfy the equation.

18. Disagree. Vertex form is best for determining maximum and minimum values, because they equal the y-coordinate of the vertex. Factored form, or standard form with technology when the quadratic relation is not factorable, is best for determining zeros.

19. I notice that the equation of the parabola is in vertex form and that the equation of the axis of symmetry is $x = 1$. I know that the vertex is on the axis of symmetry, so point (1, 1) gives me the maximum value of y: 1.

20. a) Let the origin be the far left of the diagram. Begin by determining the equation of the left stream. Its origin is at (0, 0) and its termination point is at (8, 0), since the stream is 8 m wide. These two points are its x-intercepts. I also know that in the exact centre of the pool, the streams cross at a height of 3 m. This means (5, 3) is a point on both parabolas. The factored form for the left stream is $y = ax(x - 8)$.

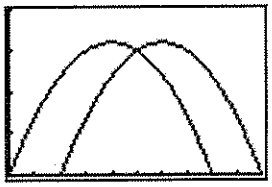
To determine a, evaluate the equation at (5, 3):

$$y = ax(x-8)$$
$$3 = a(5)(5-8)$$
$$3 = -15a$$
$$a = -\frac{1}{5}$$

The left equation is $y = -\frac{1}{5}x(x-8)$.

Since the right stream is identical in shape (the stretch factor is the same), I only need to determine the x-intercepts. The far right edge of the stream is 10 m away from 0, so 10 is a zero, and the left edge of the stream is 8 m away from 10, so it will be at $10 - 8 = 2$.

The right equation is $y = -\frac{1}{5}(x-10)(x-2)$.



b) I choose to focus solely on the left stream, since both streams have the same maximum height. Its x-intercepts are 0 and 8, which means and the equation of the axis of symmetry is $x = 4$. To determine the maximum point, I will determine the vertex by solving the equation when $x = 4$.

$$y = -\frac{1}{5}x(x-8)$$
$$y = -\frac{1}{5}(4)(4-8)$$
$$= -\frac{1}{5}(-16)$$
$$= 3.2$$

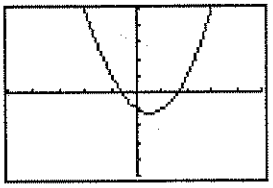
The maximum height reached by the water is 3.2 m.

Curious Math, page 296

1. long side of small rectangle : short side of small rectangle = long side of large rectangle : short side of large rectangle

2.
$$\frac{1}{x-1} = \frac{x}{1}$$
$$1 = x(x-1)$$
$$1 = x^2 - x$$
$$0 = x^2 - x - 1$$

3. For example, the corresponding quadratic relation is $y = x^2 - x - 1$. Use a graphing calculator to obtain a graph like the one below. In this graph, each tick represents one unit on the axis.



4. From question 2, the golden ratio is equal to whatever positive values of x satisfy the equation $0 = x^2 - x - 1$. The graph in question 3 is of $y = x^2 - x - 1$. Thus, when $y = 0$ in the above graph, the corresponding values of x are solutions to the equation. In other words, the zeroes of the above graph represent solutions to the equation. The golden ratio must be positive, since measurements must be positive. Therefore, the positive x-intercept on the graph represents the value of the golden ratio.

5. Use the graphing calculator to determine the value of the positive x-intercept. That value is 1.618, to three decimal places.

Lesson 5.6: Connecting Standard and Vertex Forms, page 297

1. The axis of symmetry is halfway between the x-values of points that share the same y-value:

$$x = \frac{2 + (-6)}{2}$$

$$x = -2$$

Therefore the equation of the axis of symmetry is $x = -2$.

2. Answers may vary, e.g., since the zeros are of equal distance from the axis of symmetry, factor the equation to determine the zeros. Determine two numbers whose product is $4 \times 5 = 20$ and whose sum is -12. Those numbers are -2 and -10.

$y = 4x^2 - 12x + 5$
$y = 4x^2 - 2x - 10x + 5$
$y = 2x(2x - 1) - 5(2x - 1)$
$y = (2x - 5)(2x - 1)$

$x = \frac{5}{2} = 2.5$ and $x = \frac{1}{2} = 0.5$

The points (2.5, 0) and (0.5, 0) are the same distance from the axis of symmetry.

3. $y = x(2x - 10) + 11$

$2x - 10 = 0$ and $x = 0$

$x = 5$

Since (0, 11) and (5, 11) are equal distance from the axis of symmetry, its equation is $x = 2.5$. To determine the vertex, I substitute $x = 2.5$ into the relation.

$y = 2(2.5)^2 - 10(2.5) + 11$
$y = -1.5$

The vertex is (2.5, –1.5). From the standard form, the a-value is 2. The vertex form of the equation is $y = 2(x - 2.5)^2 - 1.5$

4. a) Since (3, 0) and (7, 0) have the same y-value, I can determine the equation of the axis of symmetry by determining the midpoint.

$$x = \frac{3 + 7}{2}$$

$$= 5$$

The equation of the axis of symmetry is $x = 5$.

b) I can write $y = a(x - 3)(x - 7)$ since 3 and 7 are x-intercepts. I insert (9, –24) to determine the a-value.

$-24 = a(9 - 3)(9 - 7)$
$-24 = a(6)(2)$
$a = -2$

The equation of the parabola is $y = -2(x - 3)(x - 7)$.

When $x = 5$ then:

$y = -2(5 - 3)(5 - 7)$
$y = -2(2)(-2)$
$y = 8$

The vertex is (5, 8) and the equation in vertex form is $y = -2(x - 5)^2 + 8$.

c) To determine the standard form, I will the vertex form.

$y = -2(x - 5)^2 + 8$
$y = -2(x^2 - 10x + 25) + 8$
$y = -2x^2 + 20x - 42$

5. a) i) Answers may vary, e.g., by partial factoring, (–7, 0) and (1, 0) are two points equidistant from the axis of symmetry.

ii) Determine the axis of symmetry by determining the midpoint between (–7, 0) and (1, 0).

$$x = \frac{-7 + 1}{2}$$

$$= -3$$

The equation of the axis of symmetry is $x = -3$.

iii) When $x = -3$:

$y = (x - 1)(x + 7)$
$y = (-3 - 1)(-3 + 7)$
$y = (-4)(4)$
$y = -16$

The vertex is (–3, –16).

iv) Since the vertex is (–3, –16) and $a = 1$ (from the factored form), the equation in vertex form is $y = (x + 3)^2 - 16$.

b) i) Answers may vary, e.g., by partial factoring, two points which are equidistant from the axis of symmetry are (0, –8) and (6, –8).

ii) The axis of symmetry is midway between the x-values of (0, –8) and (6, –8).

$$x = \frac{0 + 6}{2}$$

$$= 3$$

iii) Determine the vertex by substituting in $x = 3$.

$y = x(x - 6) - 8$
$y = 3(3 - 6) - 8$
$y = 3(-3) - 8$
$y = -17$

The vertex is (3, –17).

iv) There is no constant in front of the x^2 so $a = 1$. Since $a = 1$ and the vertex is (3, – 17), then vertex form is $y = (x - 3)^2 - 17$.

c) i) Answers may vary, e.g., since the equation was given in standard form, I can use the x-intercepts as my two points; they are (–3, 0) and (7, 0).

ii) The axis of symmetry is midway between –3 and 7.

$$x = \frac{-3 + 7}{2}$$

$$= 2$$

The equation of the axis of symmetry is $x = 2$.

iii) To determine the vertex, enter $x = 2$ into the equation.

$y = -2(x + 3)(x - 7)$
$y = -2(2 + 3)(2 - 7)$
$y = -2(5)(-5)$
$y = 50$

The vertex is (2, 50).

iv) Since $a = -2$ and the vertex is (2, 50), the equation in vertex form is $y = -2(x - 2)^2 + 50$.

d) i) Answers may vary, e.g., by partial factoring, two points that are equidistant from the axis of symmetry are (0, 2) and (–4, 2).

ii) Since the axis of symmetry lies between (0, 2) and (–4, 2), determine the midpoint.

$$x = \frac{0 + (-4)}{2}$$

$$= -2$$

The equation of the axis of symmetry is $x = -2$.

iii) To determine the vertex, enter $x = -2$ into the equation.

$y = x(3x + 12) + 2$
$y = (-2)(3(-2) + 12) + 2$
$y = (-2)(6) + 2$
$y = -10$

The vertex is $(-2, -10)$.

iv) Since $a = 3$ and the vertex is $(-2, -10)$, I can write the vertex form of the equation: $y = 3(x + 2)^2 - 10$.

e) i) Answers may vary, e.g., by factoring,

$y = x^2 + 5x$
$y = x(x + 5)$
$x = 0$ and $x = -5$

Two points equidistant from the axis of symmetry are $(-5, 0)$ and $(0, 0)$.

ii) To determine the axis of symmetry, determine the midpoint.

$$x = \frac{-5 + 0}{2}$$
$$= -2.5$$

The equation of the axis of symmetry is $x = -2.5$.

iii) To determine the vertex, solve for $x = -2.5$.

$y = x^2 + 5x$
$y = (-2.5)^2 + 5(-2.5)$
$y = 6.25 - 12.5$
$y = -6.25$

The vertex is $(-2.5, -6.25)$.

iv) Since $a = 1$ and the vertex is $(-2.5, -6.25)$, the vertex form of the equation is $y = (x + 2.5)^2 - 6.25$.

f) i) Answers may vary, e.g., by partial factoring,

$y = x^2 - 11x + 21$
$y = x(x - 11) + 21$
$x = 0$ or $x = 11$

Two points equidistant from the axis of symmetry are $(0, 21)$ and $(11, 21)$.

ii) To determine the axis, I must determine the midpoint.

$$x = \frac{0 + 11}{2}$$
$$= 5.5$$

The axis of symmetry is $x = 5.5$.

iii) To determine the vertex, I solve the equation when $x = 5.5$.

$y = x^2 - 11x + 21$
$y = (5.5)^2 - 11(5.5) + 21$
$y = 30.25 - 60.5 + 21$
$y = -9.25$

The vertex is $(5.5, -9.25)$.

iv) Since $a = 1$ and the vertex is $(5.5, -9.25)$, the vertex form of the equation is $y = (x - 5.5)^2 - 9.25$.

6. Since $a = 1$, the parabola represented by the equation opens downward. Since the two graphs share a vertex, I can determine the vertex using the equation given in the question. I begin by determining two points which are equidistant from the axis of symmetry. I do this by partial factoring.

$y = x^2 - 8x + 18$
$y = x(x - 8) + 18$
$x = 0$ or $x = 8$

Two points are $(0, 18)$ and $(8, 18)$. The midpoint of the x-values is 4. Therefore, the equation of the axis of symmetry is $x = 4$. To determine the vertex, substitute $x = 4$.

$y = (4)^2 - 8(4) + 18$
$y = 16 - 32 + 18$
$y = -2$

The vertex is $(4, -2)$.

Since the equation given in the question represents the parabola that opens downward, the other one opens upward, so $a = -1$. The vertex form of that parabola is $y = -(x - 4)^2 - 2$.

7. Answers may vary, e.g.,

a) Partially factor the equation:

$y = x(x - 6) + 5$
$x = 6$ and $x = 0$

The two points are $(0, 5)$ and $(6, 5)$.

ii) The equation of the axis of symmetry is $x = \frac{0 + 6}{2} = 3$.

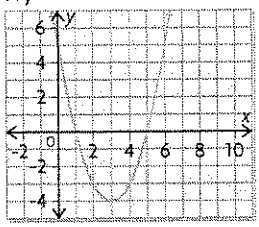
To determine the y-value of the vertex, substitute $x = 3$.

$y = x^2 - 6x + 5$
$y = (3)^2 - 6(3) + 5$
$y = 9 - 18 + 5$
$y = -4$

The vertex is $(3, -4)$.

iii) Since $a = 1$, the equation in vertex form is $y = (x - 3)^2 - 4$.

iv)



b) i) Partially factor the equation:

$y = x(x - 4) - 11$
$x = 0$ and $x = 4$

The two points are $(0, -11)$ and $(4, -11)$.

ii) The equation of the axis of symmetry is $x = \frac{0 + 4}{2} = 2$.

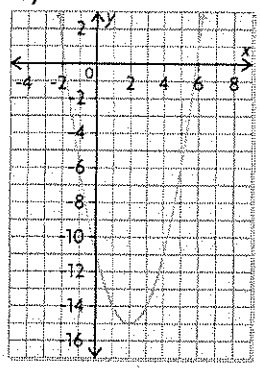
To determine the y-value, substitute $x = 2$.

$y = x^2 - 4x - 11$
$y = (2)^2 - 4(2) - 11$
$y = 4 - 8 - 11$
$y = -15$

The vertex is $(2, -15)$.

iii) Since $a = 1$, the equation in vertex form is $y = (x - 2)^2 - 15$.

iv)



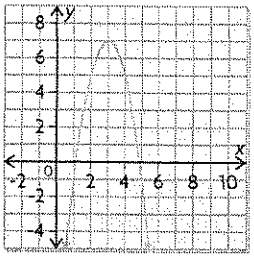
c) i) Partially factor the equation:
$y = x(-2x + 12) - 11$
$x = 0$ and $x = 6$
The two points are $(0, -11)$ and $(6, -11)$.

ii) The equation of the axis of symmetry is $x = \frac{0+6}{2} = 3$.

To determine the y-value, solve for $x = 3$.

$y = -2x^2 + 12x - 11$
$y = -2(3)^2 + 12(3) - 11$
$y = -18 + 36 - 11$
$y = 7$

The vertex is $(3, 7)$.
iii) Since $a = -2$, the equation in vertex form is $y = -2(x - 3)^2 + 7$.
iv)

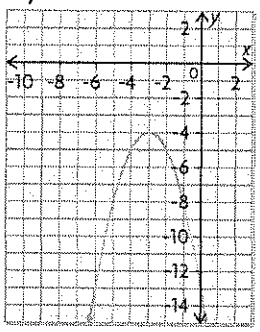


d) i) Partially factor the equation:
$y = x(-x - 6) - 13$
$x = 0$ and $x = -6$
The two points are $(0, -13)$ and $(-6, -13)$.
ii) The equation of the axis of symmetry is $x = \frac{-6+0}{2} = -3$. To determine the y-value, substitute $x = -3$.

$y = -x^2 - 6x - 13$
$y = -(-3)^2 - 6(-3) - 13$
$y = -9 + 18 - 13$
$y = -4$

The vertex is $(-3, -4)$.
iii) Since $a = -1$, the equation in vertex form is $y = -(x + 3)^2 - 4$.

iv)



e) i) Partially factor the equation:

$$y = -\frac{1}{2}x^2 + 2x - 3$$

$$y = x\left(-\frac{1}{2}x + 2\right) - 3$$

$x = 0$ and $x = 4$
The two points are $(0, -3)$ and $(4, -3)$.
ii) The equation of the axis of symmetry is $x = 2$. To determine the y-value, substitute $x = 2$.

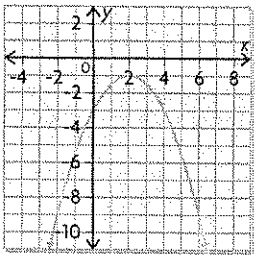
$$y = -\frac{1}{2}x^2 + 2x - 3$$

$$y = -\frac{1}{2}(2)^2 + 2(2) - 3$$

$$y = -2 + 4 - 3$$

$$y = -1$$

The vertex is $(2, -1)$.
iii) Since $a = -0.5$, the equation in vertex form is $y = -0.5(x - 2)^2 - 1$.
iv)



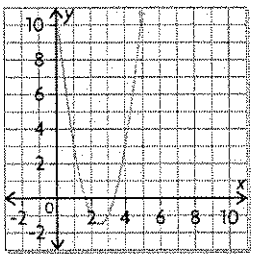
f) i) Partially factor the equation:
$y = x(2x - 10) + 11$
$x = 0$ and $x = 5$
The two points are $(0, 11)$ and $(5, 11)$.
ii) The equation of the axis of symmetry is $x = 2.5$. To determine the y-value, substitute $x = 2.5$.

$y = 2x^2 - 10x + 11$
$y = 2(2.5)^2 - 10(2.5) + 11$
$y = 12.5 - 25 + 11$
$y = -1.5$

The vertex is $(2.5, -1.5)$.

iii) Since $a = 2$, the equation in vertex form is $y = 2(x - 2.5)^2 - 1.5$.

iv)



8. Answers may vary, e.g., strategy 1: factor directly.

$y = -2x^2 + 16x - 24$
$y = -2(x^2 - 8x + 12)$
$y = -2(x - 6)(x - 2)$

The two x-intercepts are (6, 0) and (2, 0).
The axis of symmetry is at their midpoint; it is $x = 4$.
Strategy 2: use partial factoring to put relation in vertex form.

$y = -2x^2 + 16x - 24$
$y = x(-2x + 16) - 24$
$y = 0$ or $x = 8$

Two points equidistant from the axis are (0, –24) and (8, –24). The axis of symmetry in midway between them; it is $x = 4$. I prefer partial factoring since in requires less computation.

9. To determine the a-value, use the vertex form and the knowledge that the y-intercept is 7. Evaluate $y = a(x - 4)^2 - 5$ at (0, 7).

$y = a(x-4)^2 - 5$
$7 = a(0-4)^2 - 5$
$7 = 16a - 5$
$12 = 16a$
$a = \frac{3}{4}$

Go back to the original equation, insert the a-value and substitute the coordinates for the vertex to determine b.

$y = ax^2 + bx + 7$
$y = \frac{3}{4}x^2 + bx + 7$
$-5 = \frac{3}{4}(4)^2 + b(4) + 7$
$-5 = 12 + 4b + 7$
$4b = -24$
$b = -6$

The value of a is $\frac{3}{4}$ and the value of b is –6.

10. To determine the a-value, use the vertex form and the knowledge that the y-intercept is 8. Evaluate $y = a(x - 1)^2 + 7$ at (0, 8).

$y = a(x - 1)^2 + 7$
$8 = a(0 - 1)^2 + 7$
$8 = a + 7$
$a = 1$

Go back to the original equation, insert the a-value and substitute the coordinates for the vertex to determine b.

$y = ax^2 + bx + 8$
$y = x^2 + bx + 8$
$7 = 1^2 + b(1) + 8$
$7 = 1 + b + 8$
$b = -2$

The value of a is 1 and the value of b is –2.

11. To determine the maximum height, I must determine the vertex. By using partial factoring, I can determine the equation of the axis of symmetry.
$h = -5t(t - 30)$, so $t = 0$ or $t = 30$.
The equation of the axis of symmetry is $t = 15$. Now determine the height reached at 15 s by substituting $t = 15$.

$h = -5(15)^2 + 150(15)$
$= -1125 + 2250$
$= 1125$

The maximum height is 1125 m.

12. To determine the maximum height, I must determine the vertex. By using partial factoring, I can determine the equation of the axis of symmetry.
$h = t(-5t + 9) + 1$
$t = 0$ or $t = 1.8$
The axis of symmetry is $t = 0.9$. Now determine the height reached at 0.9 s by substituting $t = 0.9$.

$h = -5(0.9)^2 + 9(0.9) + 1$
$= -4.05 + 8.1 + 1$
$= 5.05$ The maximum height is 5.05 m.

13. To determine the maximum profit, I must determine the vertex. By using partial factoring, I can determine the equation of the axis of symmetry.
$P = t(-30t + 450) - 790$
$t = 0$ or $t = 15$
The equation of the axis of symmetry is $t = 7.5$. The ticket price that will get the maximum profit is $7.50.

14.a) To determine the minimum amount of gold, determine the vertex. By using partial factoring, I can determine the axis of symmetry.
$G = t(5.2t - 76) + 1492$
$t = 0$ or $t = 14.6$
The axis of symmetry is $t = 7.3$. To determine the year, add 1970 to 7.3: 1970 + 7.3 = 1977.3.
The minimum amount of gold was mined during 1977.

b) The equation of the axis of symmetry is $t = 7.3$. Determine the amount of gold mined at 7.3:

$G = 1492 - 76(7.3) + 5.2(7.3)^2$
$= 1492 - 554.8 + 277.11$
$= 5.05$

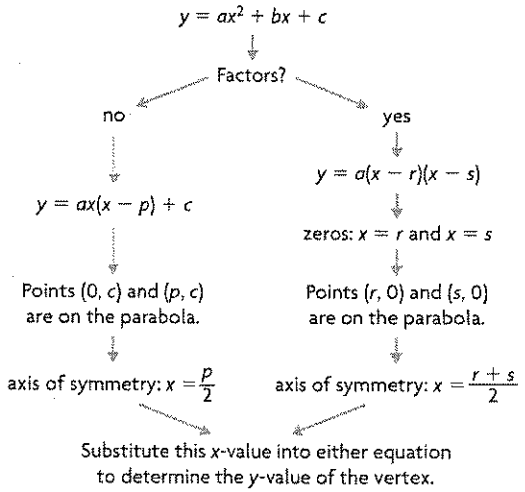
The minimum amount of gold mined was about 1215 tonnes.

c) To determine the amount of gold mined in 1985, solve the equation for $t = 1985 - 1970 = 15$.

$G = 1492 - 76(15) + 5.2(15)^2$
$= 1492 - 1140 + 1170$
$= 1522$

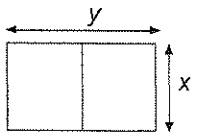
In 1985, about 1522 tonnes of gold were mined.

15.


$y = ax^2 + bx + c$
Factors?
no
yes
$y = ax(x - p) + c$
$y = a(x - r)(x - s)$
zeros: $x = r$ and $x = s$
Points (0, c) and (p, c) are on the parabola.
Points (r, 0) and (s, 0) are on the parabola.
axis of symmetry: $x = \frac{p}{2}$
axis of symmetry: $x = \frac{r + s}{2}$
Substitute this x-value into either equation to determine the y-value of the vertex.

16. Since each metre of fence costs \$5, then to determine the number of metres the farmer can afford, I have to divide 3000 by 5. The farmer can build 600 m of fencing.

Let x be the short sides of fencing and y the long side of fencing. There are three short sides and two long ones.



Then total fencing required is $3x + 2y = 600$. Isolate y:

$3x + 2y = 600$
$2y = 600 - 3x$
$y = 300 - 1.5x$

The area of the field is xy. To put this equation in terms of only one variable, I can substitute the fencing equation into the area equation:

$A = x(300 - 1.5x)$

To determine the maximum area, I need to determine the vertex. Notice that the area equation is partially factored, so (0, 0) and (200, 0) are two points which are of equal distance from the axis of symmetry. The equation of the axis of symmetry is $x = 100$. Substitute $x = 100$ into the area equation.

$A = x(300 - 1.5x)$
$= 100[300 - 1.5(100)]$
$= 100(150)$
$= 15\,000 \text{ m}^2$

The maximum area that can be enclosed is 15 000 m^2.

17. Let x be the number of \$0.05 increases. The fare after x increases will be $1.75 + 0.05x$. The number of people after x increases is shown by $9450 - 210x$. To determine the total revenue, I will multiply the number of people by the price each of them will pay:

$R = (1.75 + 0.05x)(9450 - 210x)$

To determine the maximum revenue, I will determine the vertex of the parabola. Since the equation is in factored form, I will use the x-intercepts to determine the axis of symmetry. The x-intercepts are –35 and 45, and the equation of the axis of symmetry is $x = 5$. The fare which will generate the highest revenue is $1.75 + 0.05(5) = \$2.00$. They will lose $210(5) = 1050$ riders if they use this new fare.

Chapter Review, page 304

1. a) The coefficient of x^2 must be greater than 2. Answers may vary, e.g., $y = 4x^2$, $y = 10x^2$

b) The coefficient of x^2 must be less than 0.5; it may be negative or positive. Answers may vary, e.g., $y = 0.1x^2$, $y = -0.4x^2$

c) The coefficient of x^2 must be of greater magnitude than 5 and negative. Answers may vary, e.g., $y = -6x^2$, $y = -10x^2$

2. Substitute the value of p into $y = x^2$. If the y value is less than q, then the parabola is wider than $y = x^2$; if the y value is greater than q, then the parabola is narrower than $y = x^2$.

3. a) A translation 3 units left and 4 units down means the vertex is (–3, 4). The answer is ii).

b) A translation 2 units right and 4 units down means the vertex is (2, 4). The answer is iii).

c) A translation 5 units left means the vertex is (–5, 0). The answer is iv).

d) A translation 3 units right and 2 units up means the vertex is (3, 2). The answer is i).

4. Since the graph opens downward, the a-value must be negative; this proves that c) is not a suitable choice. The vertex is (3, 8), which when entered into the vertex form produces $y = -a(x - 3)^2 + 8$. This shows that a) is not a possible solution. If the parabola was b), then when $x = 2$, $y = 5$. However, the graph shows that when $x = 2$, $y = 6$; parabola d) gives these values, so d) is the correct answer.

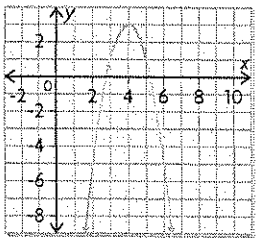
5. One similarity is both parabolas are vertically stretched by a factor of 2. They are different in that one is translated 4 units right and five units up, whereas the other is translated 5 units right and 4 units up.

6. Since rotating the parabola 180° is the same as reflecting it in the x-axis, a becomes negative. Using the vertex form, the equation is $y = -(x - 6)^2 - 8$.

7. a) He needed to stretch the graph vertically before translating it.

b) He should have started by reflecting the graph of $y = x^2$ in the x-axis; next, he should have stretched the graph vertically by a factor of 2; finally, he should have translated the graph so that its vertex moved to (4, 3).

c)



8. a) The vertex is (–5, 1). Enter this into the vertex form to get $y = a(x + 5)^2 + 1$. To determine the a-value, substitute (–8, 7), into the equation.

$$y = a(x+5)^2 + 1$$
$$7 = a(-8+5)^2 + 1$$
$$7 = 9a + 1$$
$$6 = 9a$$
$$a = \frac{2}{3}$$

The equation is $y = \frac{2}{3}(x + 5)^2 + 1$.

b) The vertex is (–1, –7). Enter this into the vertex form to get $y = a(x + 1)^2 - 7$. To determine the a-value, substitute (–2, –3) into the equation.

$$y = a(x + 1)^2 - 7$$
$$-3 = a(-2 + 1)^2 - 7$$
$$-3 = a - 7$$
$$a = 4$$

The equation is $y = 4(x + 1)^2 - 7$.

c) The vertex is (7, 0). Enter this into the vertex form to get $y = a(x - 7)^2$. To determine the a-value, substitute (9, 8) into the equation.

$$y = a(x-7)^2$$
$$8 = a(9-7)^2$$
$$8 = 4a$$
$$a = 2$$

The equation is $y = 2(x - 7)^2$.

d) The vertex is (4, 5). Enter this into the vertex form to get $y = a(x - 4)^2 + 5$. To determine the a-value, substitute (6, 7) into the equation.

$$y = a(x-4)^2 + 5$$
$$7 = a(4-6)^2 + 5$$
$$7 = 4a + 5$$
$$2 = 4a$$
$$a = \frac{1}{2}$$

The equation is $y = \frac{1}{2}(x - 4)^2 + 5$.

9. a) The vertex is (–3, 2). Enter this into the vertex form to get $y = a(x + 3)^2 + 2$. To determine the a-value, substitute (–1, 4) into the equation.

$$y = a(x+3)^2 + 2$$
$$4 = a(-1+3)^2 + 2$$
$$4 = 4a + 2$$
$$a = \frac{1}{2}$$

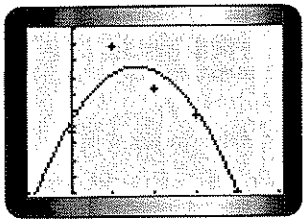
The equation is $y = \frac{1}{2}(x + 3)^2 + 2$.

b) The vertex is (1, 5). Enter this into the vertex form to get $y = a(x - 1)^2 + 5$. To determine the a-value, substitute (3, –3) into the equation.

$$y = a(x-1)^2 + 5$$
$$-3 = a(3-1)^2 + 5$$
$$-3 = 4a + 5$$
$$a = -2$$

The equation is $y = -2(x - 1)^2 + 5$.

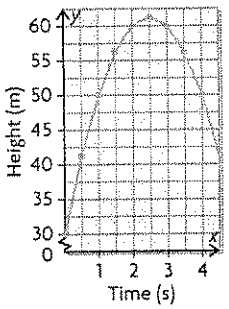
10. a)



b) Answers may vary, e.g., determine the maximum using the calculator; vertex is about (1.57, 1326). From the standard form supplied by the calculator, a is about –13. The equation is $E = -13(x - 1.57)^2 + 1326$.

c) The maximum occurs at $x = 1.57$, which is about a year and a half. To determine the precise day, multiply 0.57 by 365 to get about 208. July 27th is the 208th day of the year; therefore, the maximum occurred on about July 27th, 2003.

11. a) I plotted the points and drew the curve.



b) I estimate the vertex to be about (2.5, 61).

c) Insert the vertex into the vertex form, insert a point on the parabola, (1, 50), and determine a.

$$y = a(x - 2.5)^2 + 61$$
$$50 = a(1 - 2.5)^2 + 61$$
$$50 = 2.25a + 61$$
$$a \doteq -5$$

The equation is $-5(x - 2.5)^2 + 61$.

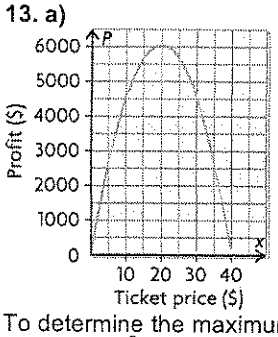
d) Answers may vary, e.g., using regression on the calculator, I got $y = -5x^2 + 25x + 30$. The standard form of relation from part c) is $y = -5x^2 + 25x + 29.75$. My estimate was extremely accurate.

12. Determine the equation of the axis of symmetry by using partial factoring:

$$y = -0.5x^2 + 1.4x + 0.1$$
$$y = -0.5x(x - 2.8) + 0.1$$

The equidistant points are (0, 0.1) and (2.8, 0.1). Thus the equation of the axis of symmetry is $x = 1.4$; this is the x-value of the vertex. Therefore 1.4 kg/ha would be required to produce the maximum yield.

13. a)



b) Partially factor to determine equidistant points:
$P = -15x^2 + 600x + 50$
$P = x(-15x + 600) + 50$
$x = 0$ and $x = 40$
Equidistant points from the axis of symmetry are (0, 50) and (40, 50). The equation of the axis of symmetry is $x = 20$.

To determine the maximum profit, substitute $x = 20$:
$P = -15(20)^2 + 600(20) + 50$
$P = 6050$
The maximum profit is \$6050 and the ticket price is \$20.

14. a) i) $y = (x - 3)(x - 5)$

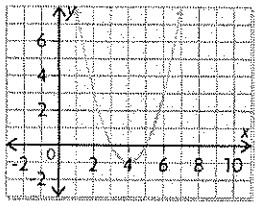
ii) The x-value of the vertex (the axis of symmetry) is halfway between 3 and 5; it is 4. Enter $x = 4$ into the equation.

$y = (4 - 3)(4 - 5)$
$y = (1)(-1)$
$y = -1$

The vertex is $(4, -1)$.

iii) From the standard form, $a = 1$. The vertex is $(4, -1)$, so the equation in vertex form is $y = (x - 4)^2 - 1$.

iv)



b) i) $y = 2x^2 - 8x - 64$
$y = 2(x^2 - 4x - 32)$
$y = 2(x + 4)(x - 8)$

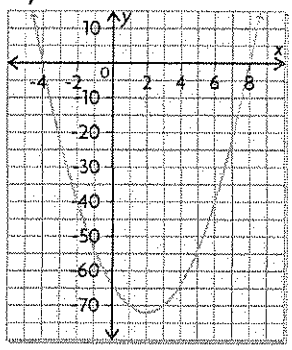
ii) The x-value of the vertex (the axis of symmetry) is halfway between -4 and 8; it is 2. Insert $x = 2$ and solve.

$y = 2(2 + 4)(2 - 8)$
$y = 2(6)(-6)$
$y = -72$

The vertex is $(2, -72)$.

iii) Partially factor to determine points equidistant from the axis of symmetry:
$y = 2x^2 - 8x - 64$
$y = x(2x - 8) - 64$
$x = 0$ and $x = 4$
The equidistant points are $(0, -64)$ and $(4, -64)$. The equation of the axis of symmetry is therefore $x = 2$. Substitute into the standard form to determine the y-value of the vertex:
$y = 2(2)^2 - 8(2) - 64$
$y = -72$
From the standard form, $a = 2$. The equation in vertex form is $y = 2(x - 2)^2 - 72$.

iv)



c) i) $y = -4x^2 - 12x + 7$
$y = -(4x^2 + 12x - 7)$
$y = -(2x + 7)(2x - 1)$

ii) The zeros are $(-3.5, 0)$ and $(0.5, 0)$; the equation of the axis of symmetry is $x = \dfrac{-3.5 + 0.5}{2} = -1.5$

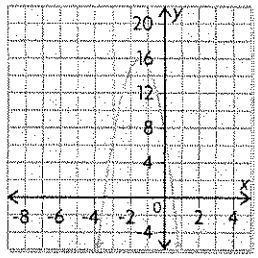
Insert $x = -1.5$ into the equation and solve.

$y = -[2(-1.5) + 7][2(-1.5) - 1]$
$y = -(4)(-4)$
$y = 16$

The vertex is $(-1.5, 16)$.

iii) Partially factor to determine points equidistant from the axis of symmetry:
$y = -4x^2 - 12x + 7$
$y = x(-4x - 12) + 7$
$x = 0$ and $x = -3$ The points are $(0, 7)$ and $(-3, 7)$.
The equation of the axis of symmetry is therefore $x = -1.5$. From the standard form, $a = -4$. The equation in vertex form is $y = -4(x + 1.5)^2 + 16$.

iv)



15. Answers may vary, e.g.,

a) $y = x^2 + 2x + 5$
$y = x(x + 2) + 5$

Two points are $(0, 5)$ and $(-2, 5)$. The equation of the axis of symmetry is $x = \dfrac{-2 + 0}{2} = -1$. To determine the y-value of the vertex, evaluate the equation at $x = -1$.

$y = (-1)^2 + 2(-1) + 5$
$y = 1 - 2 + 5$
$y = 4$

From the standard form, $a = 1$.
The equation in vertex form is $y = (x + 1)^2 + 4$.

b) $y = -x^2 + 6x - 3$
$y = -x(x - 6) - 3$
Two points are (0, –3) and (6, –3). The equation of the axis of symmetry is $x = \frac{0+6}{2} = 3$. To determine the vertex, evaluate the equation at $x = 3$.
$y = -(3)^2 + 6(3) - 3$
$y = -9 + 18 - 3$
$y = 6$
From the standard form, $a = -1$.
The equation in vertex form is $y = -(x - 3)^2 + 6$.
c) $y = -3x^2 + 42x - 147$
$y = x(-3x + 42) - 147$
Two points are (0, –147) and (14, –147). The equation of the axis of symmetry is $x = \frac{0+14}{2} = 7$. To determine the vertex, evaluate the equation at $x = 7$.
$y = -3(7)^2 + 42(7) - 147$
$y = -147 + 294 - 147$
$y = 0$
From the standard form, $a = -3$.
The equation in vertex form is $y = -3(x - 7)^2$.
d) $y = 2x^2 - 20x + 41$
$y = x(2x - 20) + 41$
Two points are (0, 41) and (10, 41). The equation of the axis of symmetry is $x = \frac{0+10}{2} = 5$. To determine the vertex, evaluate the equation at $x = 5$.
$y = 2(5)^2 - 20(5) + 41$
$y = 50 - 100 + 41$
$y = -9$
From the standard form, $a = 2$.
The equation in vertex form is $y = 2(x - 5)^2 - 9$.

16. a) Use partial factors.
$y = x^2 - 6x - 8$
$y = x(x - 6) - 8$
Find two points with a y-coordinate of –8 by setting each partial factor equal to 0.
$x = 0$ or $x - 6 = 0$
So, $x = 0$ or $x = 6$.
The points (0, –8) and (6, –8) are on the parabola.
The equation of the axis of symmetry is found by calculating the mean of the x-coordinates of these points.
$x = \frac{0+6}{2} = 3$
Since the parabola is symmetric, the vertex is on the line $x = 3$.
Substitute this value into the relation.
$y = x^2 - 6x - 8$
$y = 3^2 - 6(3) - 8$
$y = 9 - 18 - 8$
$y = -17$
The vertex of the parabola is at (3, –17).
From the standard form, $a = 1$.
The vertex form of the equation is $y = (x - 3)^2 - 17$.
b) By using the x-intercepts: The x-intercepts are –3 and 7, and since the vertex is in the middle of these two points, then its x-coordinate is $x = \frac{-3+7}{2} = 2$. Determine the y-value by substituting $x = 2$.
$y = -2(x + 3)(x - 7)$
$y = -2(2 + 3)(2 - 7)$
$y = -2(5)(-5)$
$y = -50$
The vertex is (2, –50), and the a-value is –2.
The equation in vertex form is $y = -2(x - 2)^2 + 50$.
c) Use partial factors. $y = x(3x + 12) + 2$
Find two points with a y-coordinate of 2 by setting each partial factor equal to 0.
$x = 0$ or $3x + 12 = 0$
$x = 0$ or $x = -4$.
The points (0, 2) and (–4, 2) are on the parabola.
The equation of the axis of symmetry is found by calculating the mean of the x-coordinates of these points.
$x = \frac{0+(-4)}{2}$
$= -2$
Since the parabola is symmetric, the vertex is on the line $x = -2$. Substitute this value into the relation.
$y = x(3x + 12) + 2$
$y = -2[3(-2) + 12] + 2$
$y = -2(6) + 2$
$y = -10$
The vertex of the parabola is at (–2, –10).
From the standard form, $a = 3$.
The equation in vertex form is $y = 3(x + 2)^2 - 10$.
d) Use partial factors.
$y = -2x^2 + 12x - 11$
$y = -2x(x - 6) - 11$
Find two points with a y-coordinate of –11 by setting each partial factor equal to 0.
$x = 0$ or $x - 6 = 0$
$x = 0$ or $x = 6$.
The points (0, –11) and (6, –11) are on the parabola.
The equation of the axis of symmetry is found by calculating the mean of the x-coordinates of these points.
$x = \frac{0+6}{2}$
$= 3$
Since the parabola is symmetric, the vertex is on the line $x = 3$. Substitute this value into the relation.
$y = -2x^2 + 12x - 11$
$y = -2(3)^2 + 12(3) - 11$
$y = -18 + 36 - 11$
$y = 7$
The vertex is at (3, 7). From the standard form, $a = -2$.
The equation in vertex form is $y = -2(x - 3)^2 + 7$.

17. a) Since the initial height is the y-intercept, then in standard form, I can read off the constant value. The height of the football when it was kicked was 1.1 m.
b) Use partial factors to determine the vertex.
$h = -4.9t^2 + 22.54t + 1.1$
$h = -4.9t(t - 4.6) + 1.1$
Determine two points with a h-coordinate of 1.1 by setting each partial factor equal to 0.
$t = 0$ or $t - 4.6 = 0$
So, $t = 0$ or $t = 4.6$. The points (0, 1.1) and (4.6, 1.1) are on the parabola. The equation of the axis of symmetry is found by calculating the mean of the t-coordinates of these points.

$$t = \frac{0 + 4.6}{2}$$
$$= 2.3$$

Since the parabola is symmetric, the vertex is on the line $t = 2.3$. Substitute this value into the relation.

$h = -4.9t^2 + 22.54t + 1.1$
$h = -4.9(2.3)^2 + 22.54(2.3) + 1.1$
$h \doteq 27.0$

The vertex of the parabola is at (2.3, 7).
The maximum height is 27.0 m at a time of 2.3 s.

Chapter Self-Test, page 306

1. a) I used the vertex form since I knew the new vertex was (1, –9). The equation is of the form $y = a(x - 1)^2 - 9$. The point (5, –1) is on the graph, so use that to determine the value of a.

$y = a(x-1)^2 - 9$
$-1 = a(5-1)^2 - 9$
$-1 = 16a - 9$
$8 = 16a$
$a = \frac{1}{2}$

The new equation is $y = \frac{1}{2}(x-1)^2 - 9$.

b) The black parabola has been vertically compressed by a factor of $\frac{1}{2}$, translated 1 unit right and translated 9 units down.

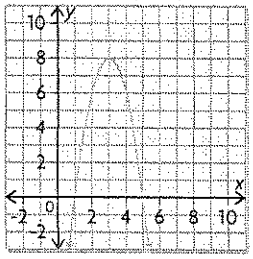
2. a) A parabola that opens downward has a negative a. Then, we know $a = -4$. Inserting this information into the vertex form, we get $y = -4(x - 7)^2 + 5$.

b) The x-value of the vertex is between the two x-intercepts, and the y-value is the minimum point. The x-value between 1 and 5 is 3, and the y-value is –12. Insert the vertex, (3, –12), into the equation. Now we have $y = a(x - 3)^2 - 12$. Substitute (6, 15) and solve for a.

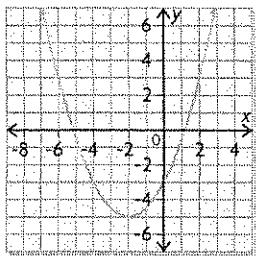
$y = a(6 - 3)^2 - 12$
$15 = 9a - 12$
$9a = 27$
$a = 3$

The equation of the parabola is $y = 3(x - 3)^2 - 12$.

3. a) First, apply the reflection through the x-axis; then, apply the vertical compression of factor 2, and translate the parabola to (3, 8).



b) First, apply the vertical stretch factor of 0.5. Then, translate the parabola to (–2, –5).



4. For example, since (4, –10) is down and to the right of the original vertex, let's choose it to be the new vertex. In vertex form, the equation is $y = a(x - 4)^2 - 10$. Since there is a vertical compression, a needs to be greater than 1. Let a be 2. A possible equation is $y = 2(x - 4)^2 - 10$.

5. a) $P = -2x^2 + 24x - 54$
$P = -2(x^2 - 12x + 27)$
$P = -2(x - 3)(x - 9)$

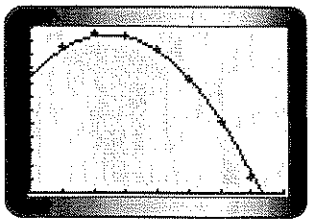
b) The zeros are 3 and 9. They represent the break-even values in $100 000s.

c) The maximum profit is at the vertex of the parabola. The x-value of the vertex is halfway between the x-intercepts; in this case it is 6. I can now use the x-value to determine the y-value.

$P = -2(6 - 3)(6 - 9)$
$P = -2(3)(-3)$
$P = 18$

The number of shoes sold is 600 000 and the maximum profit is $1 800 000.

6. a)



b) I estimate the vertex to be (2.5, 115). Using this vertex, my equation would be $h = a(t - 2.5)^2 + 115$. To determine a, substitute a point on the curve in the equation. Use (6, 52).

$h = a(t - 2.5)^2 + 115$
$52 = a(6 - 2.5)^2 + 115$
$52 = a(12.25) + 115$
$a \doteq -5$

b) I estimate the vertex to be (2.5, 115). Using this vertex, my equation would be $h = a(t - 2.5)^2 + 115$. To determine a, substitute a point on the curve into the equation. Use (6, 52).

$h = a(t - 2.5)^2 + 115$
$52 = a(6 - 2.5)^2 + 115$
$52 = a(12.25) + 115$
$a \doteq -5$

The equation is $h = -5(t - 2.5)^2 + 115$ in vertex form. To determine the standard form, I expand it.

$h = -5(t - 2.5)^2 + 115$
$= -5(t^2 - 5t + 6.25) + 115$
$= -5t^2 + 25t + 83.75$

c) Answers may vary, e.g., from my calculator, I get a regression equation of $h = -5t^2 + 24t + 88$. This is very close to my model in part b).

d) To determine the maximum height, I have to determine the vertex of $h = -5t^2 + 24t + 88$. I will do so by using partial factoring.

$h = -5t^2 + 24t + 88$
$h = t(-5t - 24) + 88$

$t = 0$ or $t = -\frac{24}{5} = -4.8$

These values are of equal distance from the axis of symmetry, so use their mean to determine its equation:

$$t = \frac{0 - (-4.8)}{2} = 2.4$$

Substitute this value in the equation to obtain the y-coordinate of the vertex:

$h = -5(2.4)^2 + 24(2.4) + 88$
$h = -28.8 + 57.6 + 88$
$h = 116.8$

The maximum height is 116.8 m at time 2.4 s.

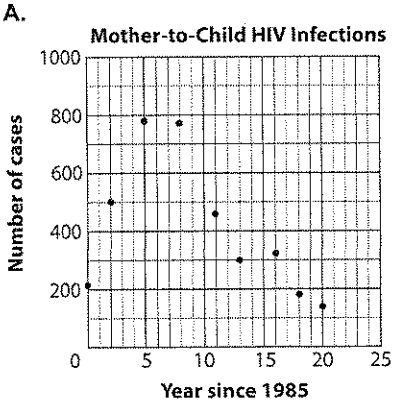
e) To determine the time the rocket will hit the ground, I let $h = 0$ and solve for t.

$$\begin{aligned} -5(t-2.4)^2 + 116.8 &= 0 \\ 5(t-2.4)^2 &= 116.8 \\ (t-2.4)^2 &= 23.36 \\ t-2.4 &= 4.833 \\ t &= 7.23 \end{aligned}$$

The rocket will hit the ground at about 7.23 s after it was launched.

Chapter Task, page 307

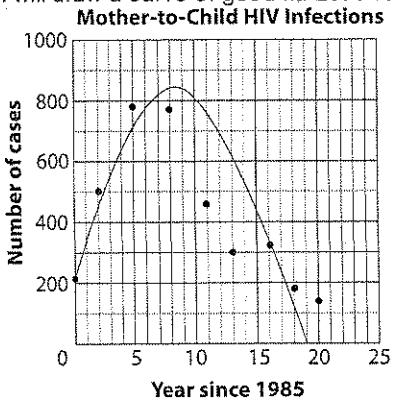
A.



Using a quadratic model makes sense because the shape formed by these points appears to match that of a parabola. A line would not make a good model because the points form an upside-down U.

B. I could draw a curve of good fit, estimate the zeros, and use the factored form. Or, I could draw a curve of good fit, estimate the vertex, and use the vertex form. I would then substitute a point to determine the value of a. I can also use graphing technology to plot the points, perform a quadratic regression, and obtain an equation in standard form.

I will draw a curve of good fit. Let 0 represent 1985.



I estimate the vertex to be about (8.5, 840).

Substituting the vertex into the vertex form, I get:

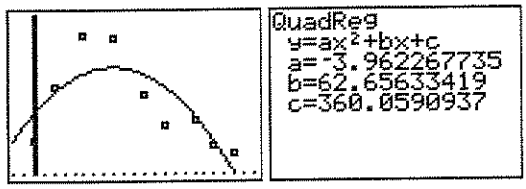
$$y = a(x-8.5)^2 + 840$$

To determine a, I substitute point (0, 210), which is on the parabola, into the equation:

$$\begin{aligned} y &= a(x-8.5)^2 + 840 \\ 210 &= a(0-8.5)^2 + 840 \\ 210 &= 72.25a + 840 \\ -630 &= 72.25a \\ a &\doteq -8.72 \end{aligned}$$

My equation is $y = -8.72(x-8.5)^2 + 840$.

If I used a graphing calculator, my solution would be as follows:

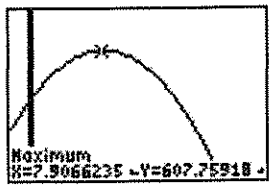


The equation from the quadratic regression is, with the coefficients rounded to three places,

$y = -3.962x^2 + 62.656x + 360.059$.

C. The parabola has been reflected across the x-axis, translated up 840 units, translated right 8.5 units, and stretched vertically by a factor of 8.72.

From my calculator, $a = -3.962$. I can use the calculator to determine the vertex of the relation.

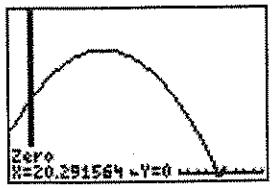


The vertex is (7.907, 607.759). This information tells me that the parabola was stretched vertically by a factor of 3.962, reflected through the x-axis, and translated 7.907 units right and 607.759 units up.

D. The introduction of an effective drug therapy occurred when the number of infections started decreasing. This occurs at the vertex. Base on my curve of good fit, this occurs at about $x = 8$, or 1993. On the calculator, the vertex is at about $x = 7.9$, which corresponds to the end of 1992.

E. Based on my curve of good fit, the number of cases will reach zero in about year 19, or 2004.

With my calculator, I used the Zero function to determine when the value was 0.



The zero occurred at about $x = 20$, or the year 2005. According to the parabola I chose, HIV cases should have been 0 by 2005, but the data show otherwise.

If I look at the scatter plot, I can see that there is a steep decline at first, but then the trend levels out. This leads me to believe that it will take much longer for the number of HIV infections to be reduced to zero than my model predicts.

F. By being able to estimate the number of people infected with HIV, the government can develop effective treatment plans, and it can ensure that there is enough medication available. Mathematical descriptions also allow agencies to determine trends. For example, if there is a downward trend, then a treatment is working; if there is not, then a different protocol should be tried. Also, mathematics can help to determine the most at-risk age group so that preventative measures can be targeted toward them.